

Performance and Quality Analysis of Interpolation Methods for Coupling

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Outline

- Motivation
- Coupling Approaches
 - Interpolation/Evaluation
- Results of Investigation
- Summary

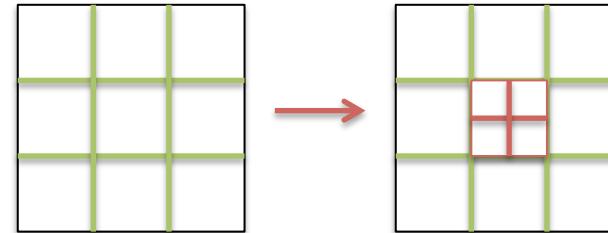
Motivation

- More and more complex devices
 - Better understanding of the physical phenomena
 - Improvements in product design in the engineering field
- Important:
Applications involve multi-physics and multi-scales



Strategy

- Simulating whole problem with one solver
→ too expensive
- **Partitioned coupling**
 - splitting whole domain in subdomains
 - according physical phenomenon
 - Solving each domain according to its requirements
 - equations, element size etc.



Partitioned Coupling

FASTESt
Open  FOAM
Navier-Stokes
equations
FV 2nd order

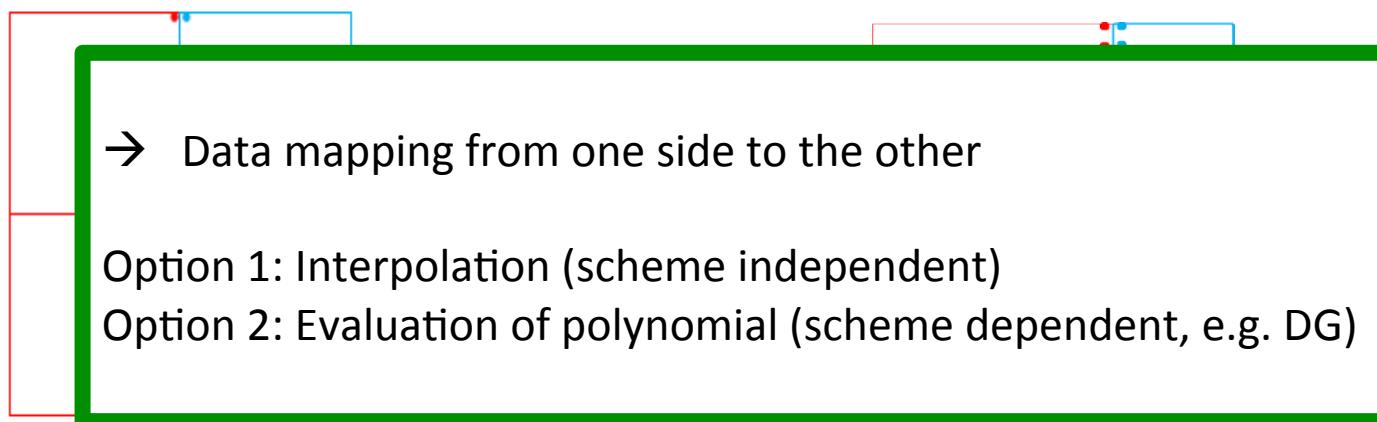

Euler equations
DG 4-8th order


Linearized Euler
equations
DG 8-64th order

- Coupling different domains with different numerical methods (Finite Volume, Discontinuous Galerkin) e.g. FSA-Interaction
- **Coupled simulation with spatial discretization**

Coupling Types

Matching



Non-matching

→ Matching coupling

- same grid size
- same scheme order

→ same number of coupling points

→ Non-matching coupling

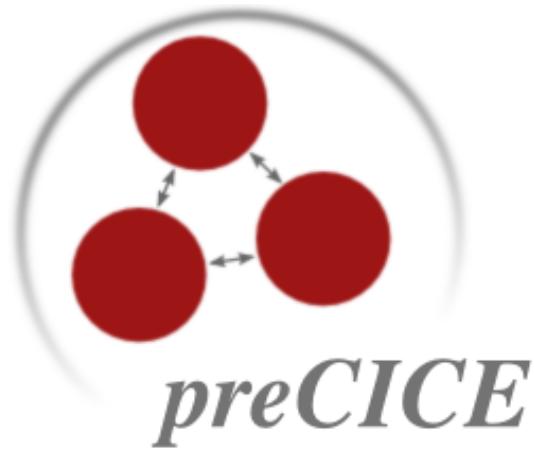
- different grid size
- different scheme order

→ different number of coupling points

Two coupling approaches

1) preCICE

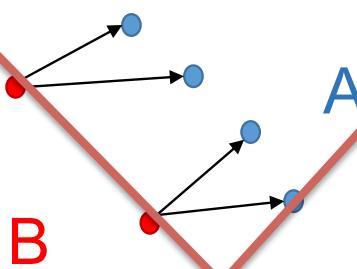
- An external library
- Treats solver as a black-box
- Different coupling strategies
 - Parallel/ serial explicit, Implicit quasi newton methods
- Different data-mapping methods
 - Nearest-Neighbor, Nearest-Projection, Radial-Basis-Function



B. Gatzhammer, B. Uekermann (München),
F. Lindner, M. Mehl (Stuttgart)

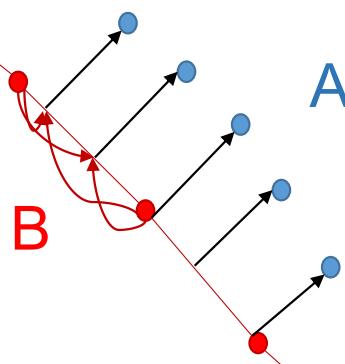
Interpolation Methods

1a) Nearest-Neighbour



- + No computation
- 1st order accuracy
- not suitable for non-matching coupling

1b) Nearest-Projection



- + 2nd order accuracy
- provide neighborhood information

1c) Radial-Basis-Function

$$s(x) = \sum_{i=1}^N \gamma_i \cdot \varphi(\|x - x_i\|) + q(x)$$

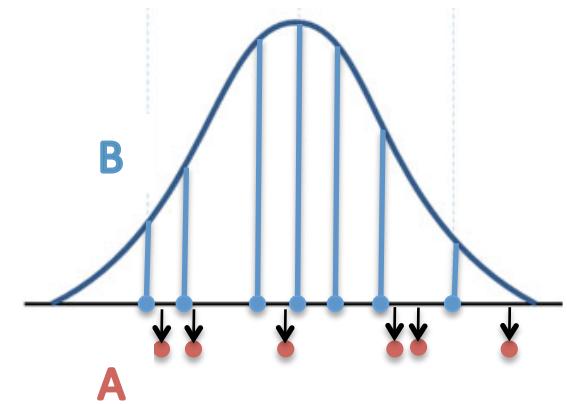
φ : basis function
 x_i : point, which has to be evaluated
 $q(x)$: global linear function
 $s(x)$: interpolant

- + No neighborhood information needed
- + Different basis-functions available
- Linear equation system has to be solved
- Condition number depends on point distribution
(equidistant/non-equidistant)

e.g. basis function: Gaussian function

$$\text{shape-parameter: } a = \frac{\sqrt{-\ln(10^{-9})}}{m \cdot h}$$

m : average number of points to cover
 h : average distance between points



Two coupling approaches

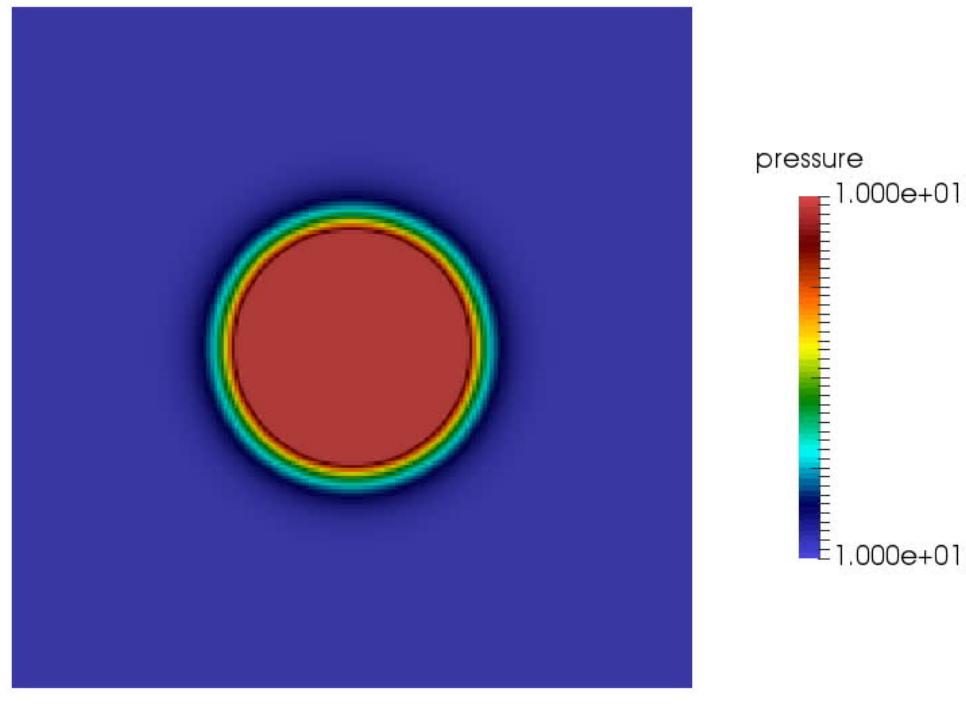
2) APESmate

- Integrated approach in APES
- Evaluation of polynomial
- Data-mapping according to scheme order
- Accuracy depends on scheme order

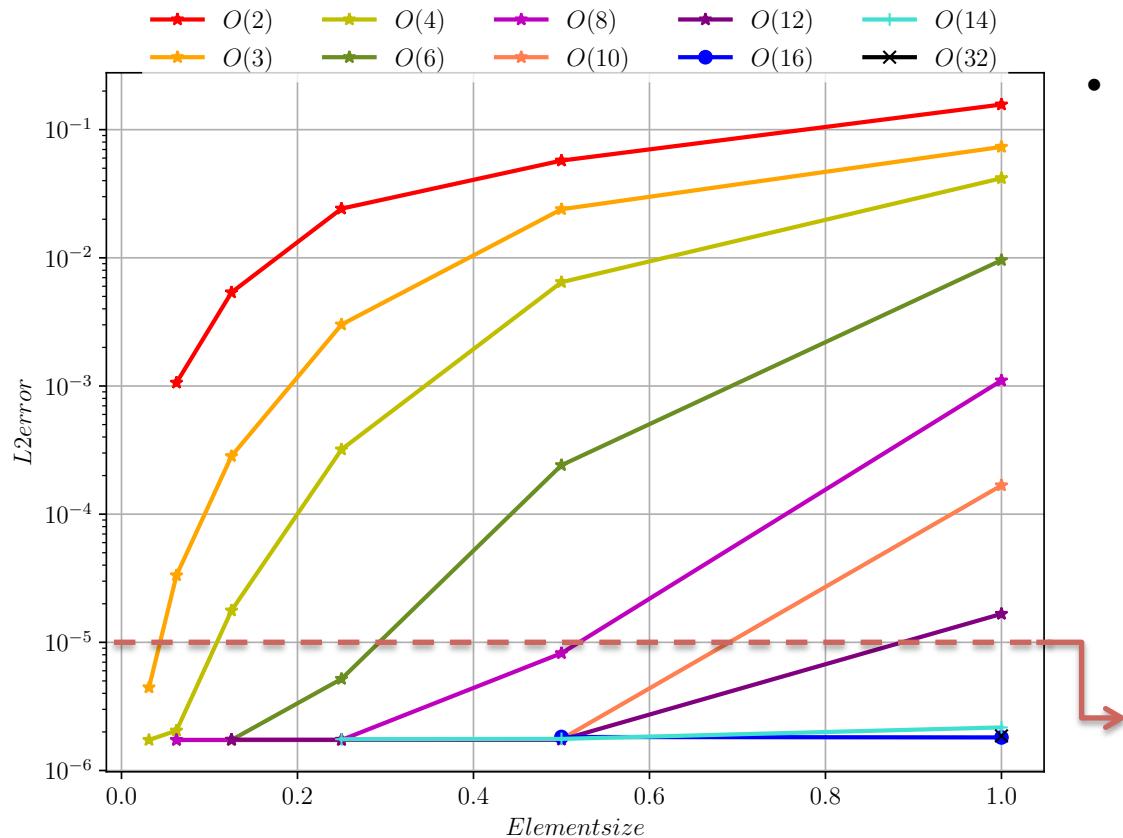
K. Masilamani, V. Krupp, H. Klimach, S. Roller (Siegen)



Results: Testcase



Results: Convergence Study



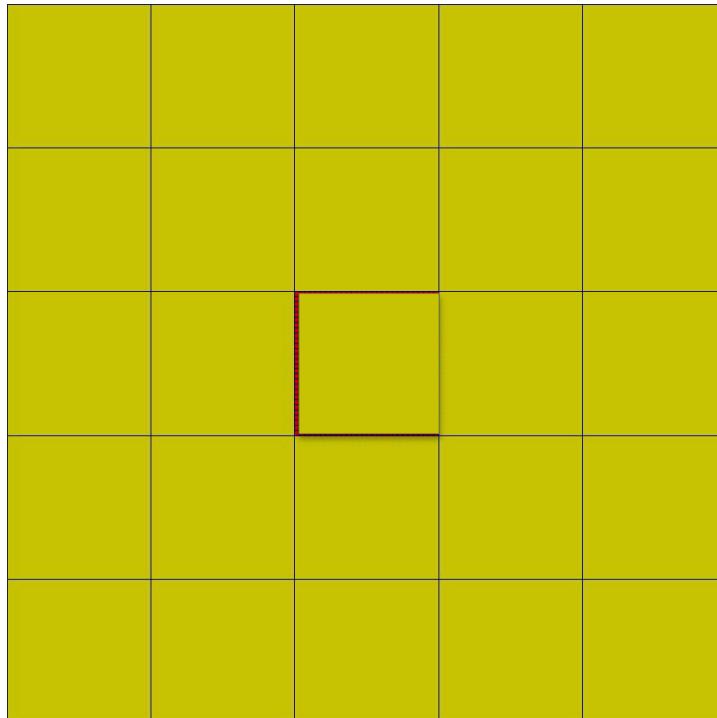
- Monolithic simulation
 - Variation of the element size and scheme order
 - Calculation of the L2error from simulation and analytical solution

Chosen element size /
L2error for coupling two
domains

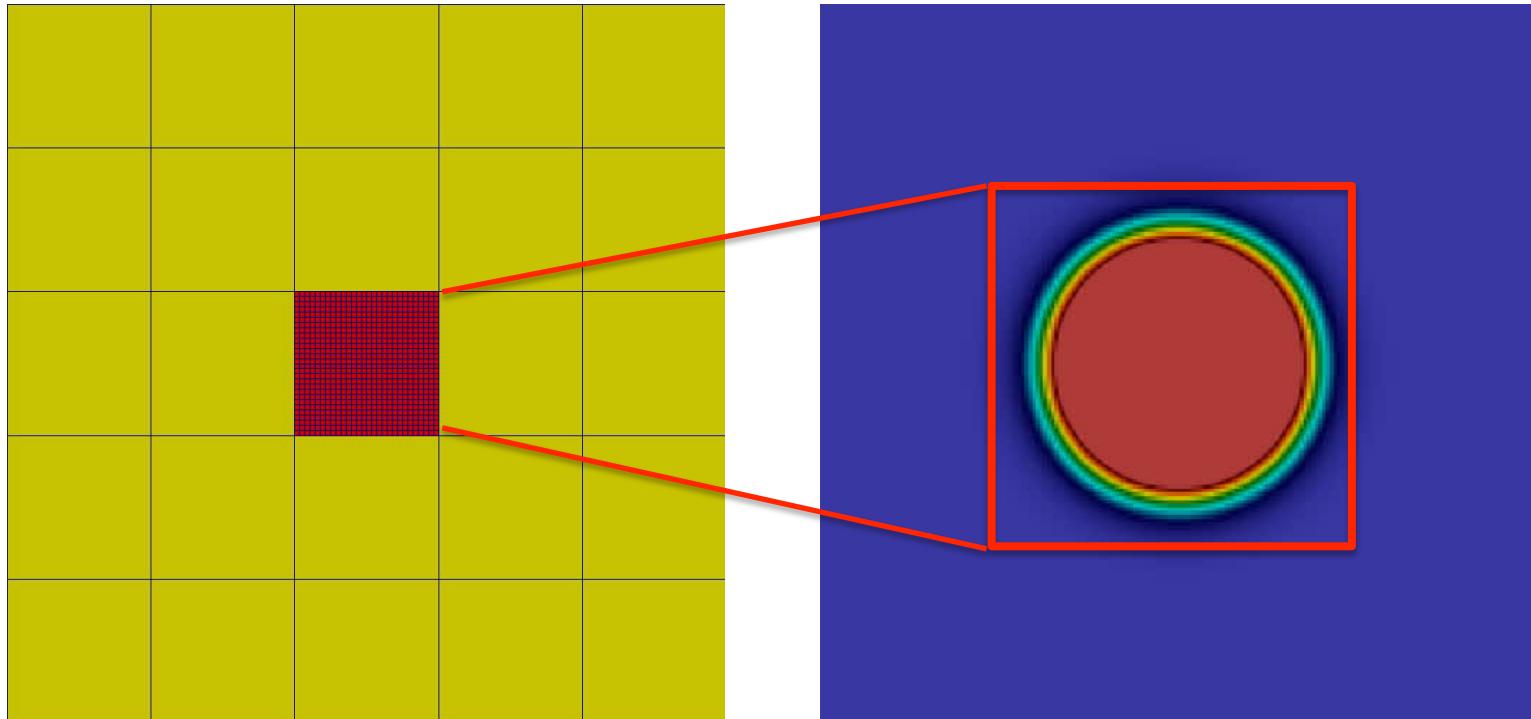
Testcases:

	Scheme Order	Number of Points	Number of Elements
Testcase 1 Inner: Outer:	O(3) O(4)	1728 960	32768 124000
Testcase 2 Inner: Outer:	O(3) O(6)	1728 864	32768 7936
Testcase 3 Inner: Outer:	O(3) O(8)	1728 768	32768 992
Testcase 4 Inner: Outer:	O(3) O(14)	1728 1176	32768 124

Testcase: Gaussian Pressure Pulse



Testcase: Gaussian Pressure Pulse



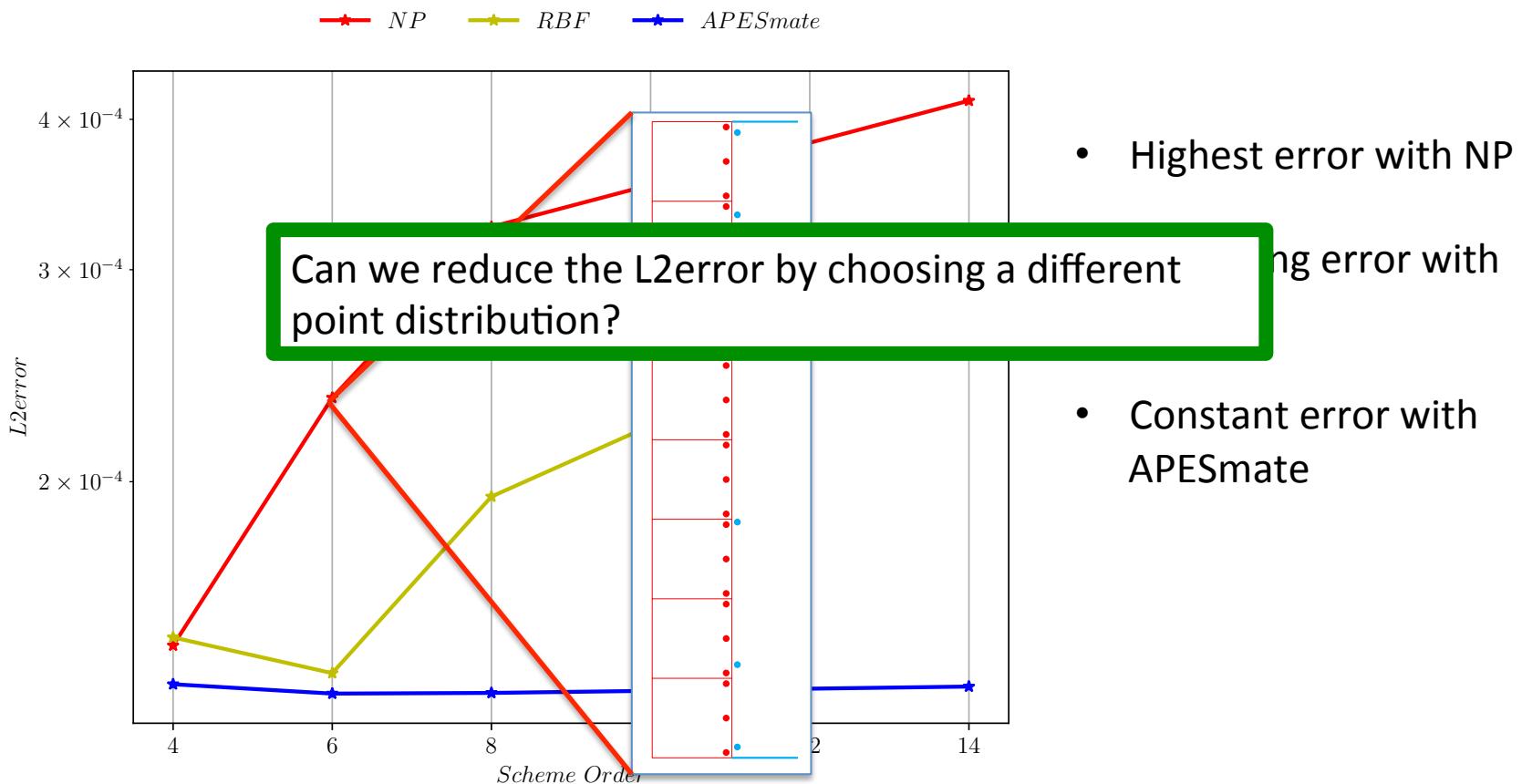
Domain: $5 \times 5 \times 5$

Inner domain: $1 \times 1 \times 1$

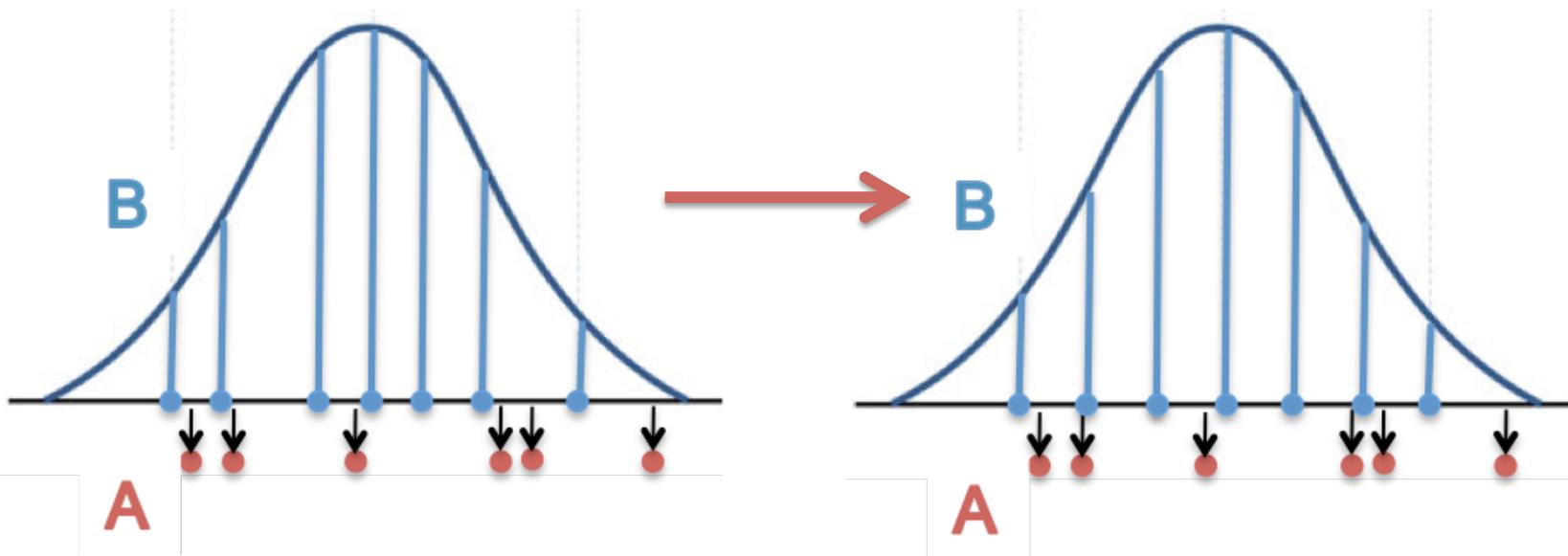
Halfwidth: 0.25

Amplitude: 1.0

Results: Interpolation/Evaluation method

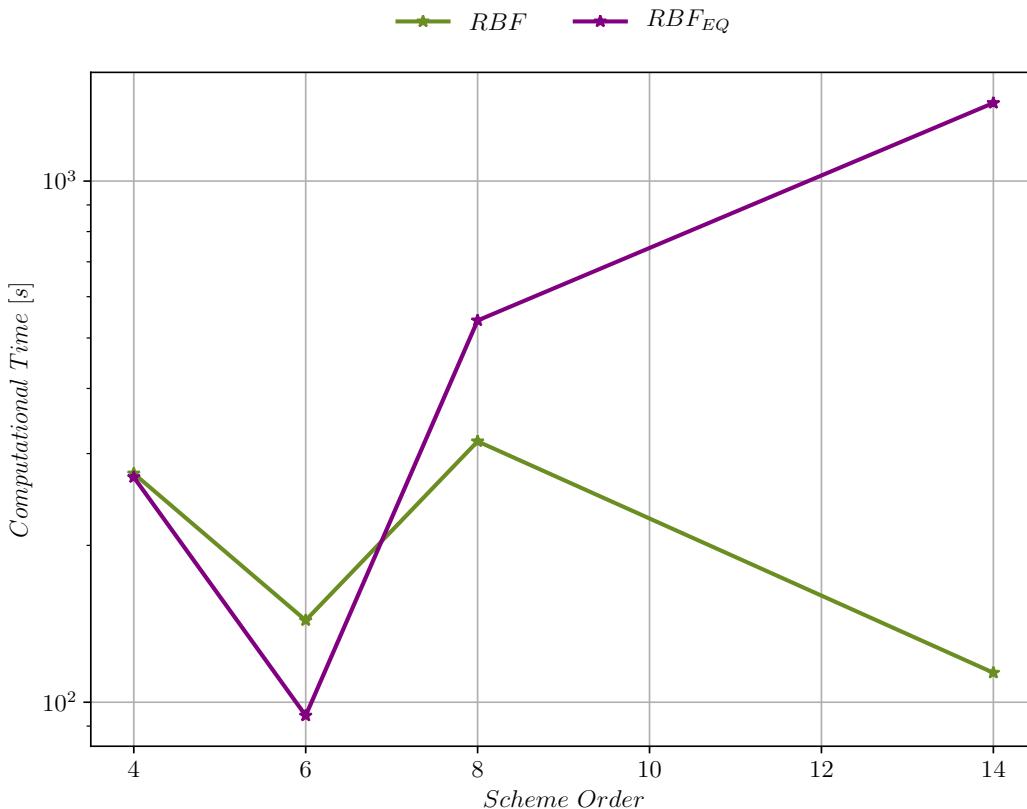


Results: Equidistant Points for Radial-Basis-Function



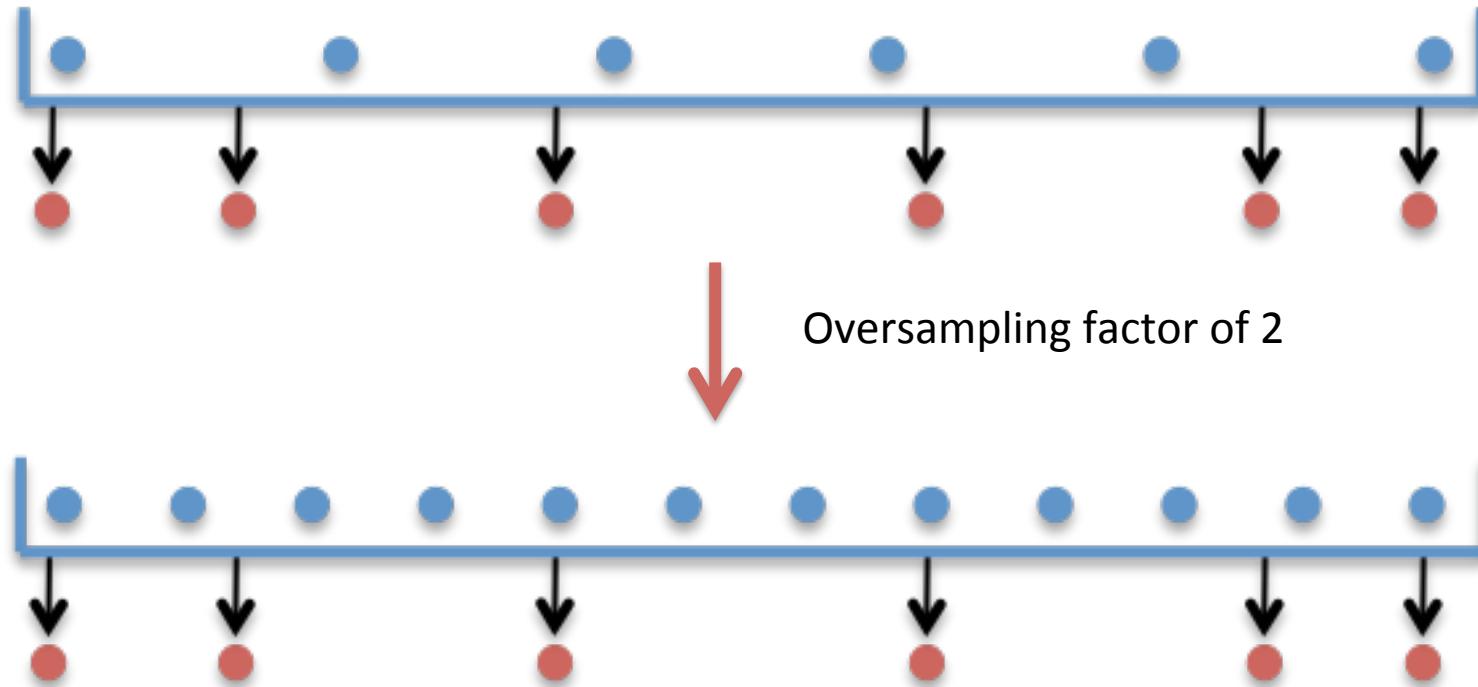
Providing equidistant points for the
interpolation

Results: Radial-Basis-Function

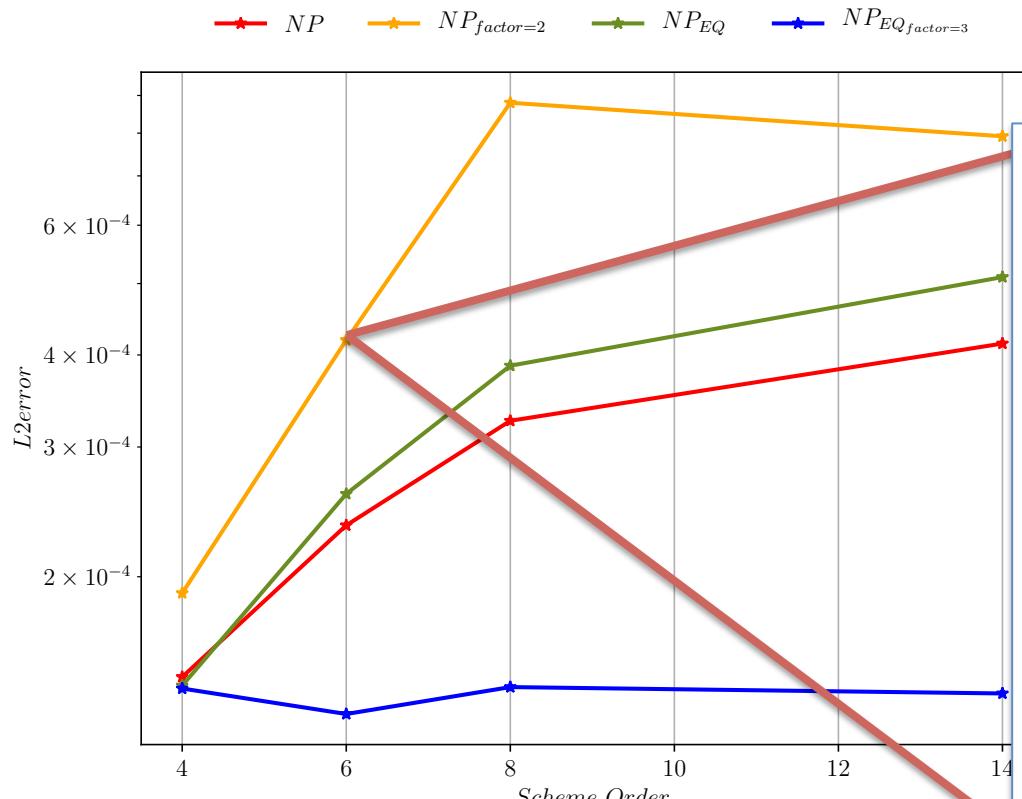


- Increasing error for RBF with non-equidistant points
- Decreasing error with equidistant point distribution
- Increasing computational cost for RBF with equidistant points

Results: Oversampling for Nearest-Projection

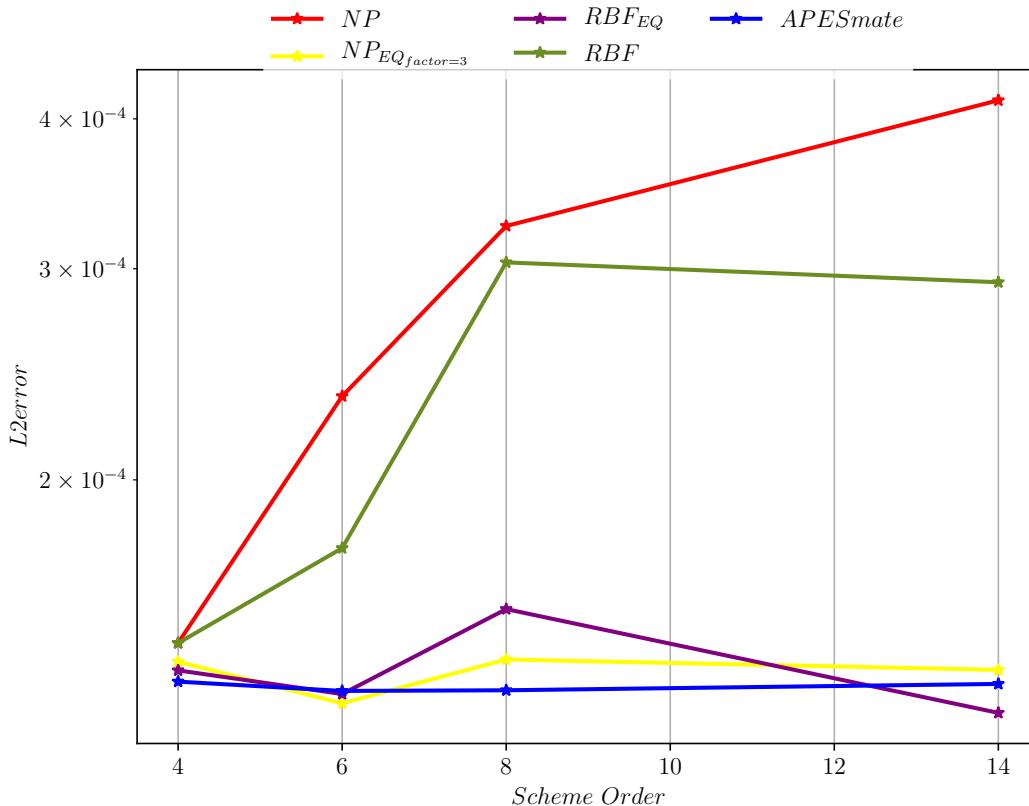


Results: Nearest-Projection



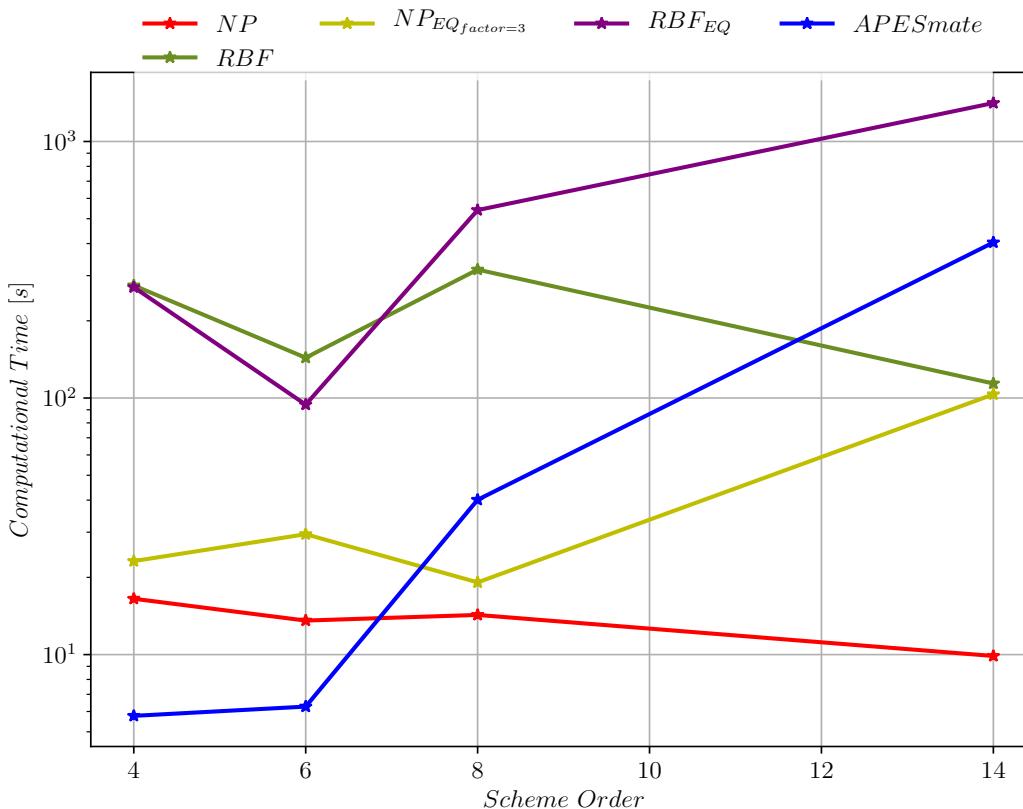
- Increasing error, when using non-coupling for right points
- Decreasing error with increasing factor of sampling points

Results: L2error for all methods



- Highest error with NP and non-equidistant points
- Error APESmate and NP almost constant over scheme order
- RBF with equidistant points → decreasing error

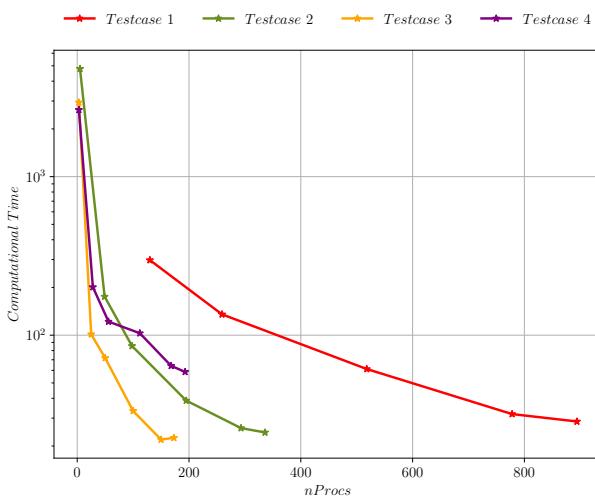
Results: Computational cost



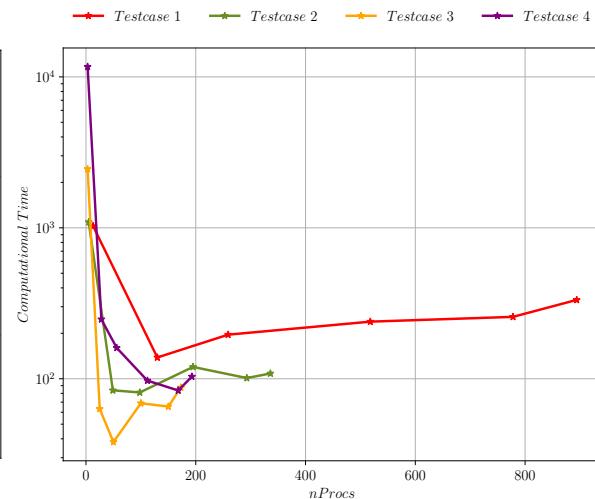
- RBF with equidistant points → highest cost
- NP with non-equidistant points → lowest cost
- APESmate → increasing cost with increasing scheme order

Results: Performance (Weak Scaling)

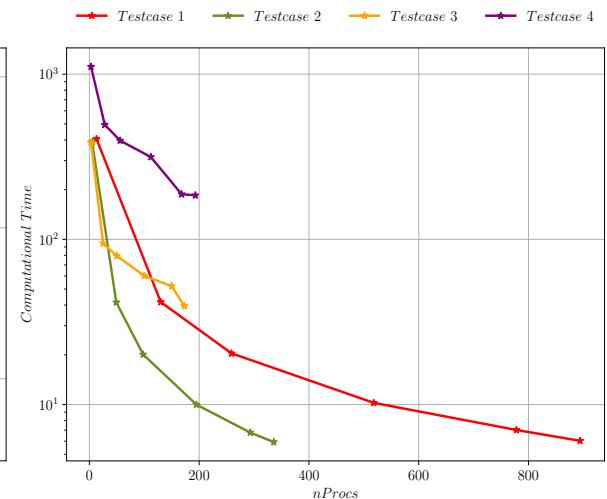
Nearest-Projection



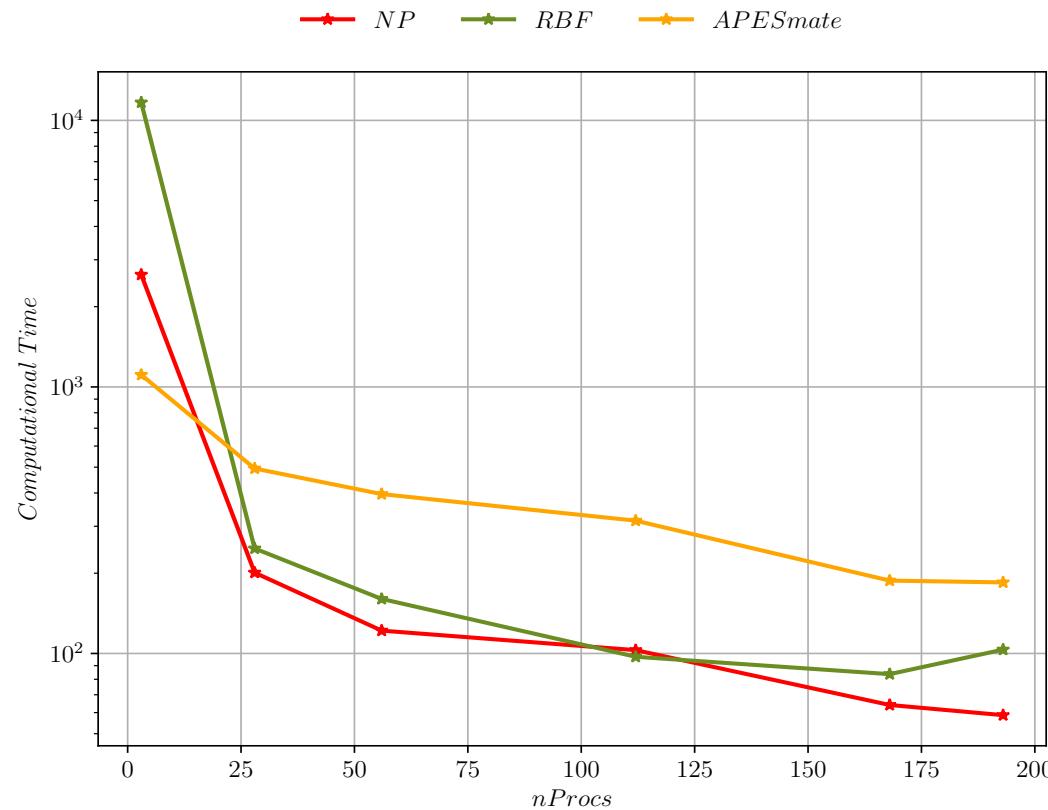
Radial-Basis-Function



APESmate



Results: Performance (Weak Scaling)



Summary

Coupling Tool	Method	Error	Computational cost
preCICE	RBF non-equidistant points	Increase with scheme order	Increasing with scheme order
		<ul style="list-style-type: none">• When coupling different solvers → preCICE with Nearest-Projection interpolation• When using our solvers → APESmate provides very good results	
	NP equidistant points	Decrease with oversampling factor	Increasing with scheme order and oversampling factor
APESmate	Evaluation of the polynomial	Scheme dependent	Increasing with scheme order

Thank you for your attention