Data compression strategies for exascale CFD simulations

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Outline

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Motivation

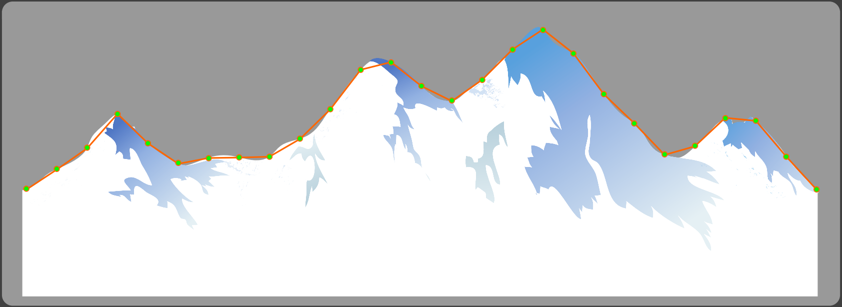
- Move towards exascale computing due to increasingly complex demands on scientific and numerical modelling.
- Growing mismatch between the ability to produce and store/analyze data.
- Alleviating the I/O bottleneck in exascale computing by considerable data-reduction before I/O.
- Image and video compression algorithms offer robust and portable source codes with a wide range of compression ratios (lossy/lossless).

Outline

1. Motivation
2. Image Compression Overview
   - Reduction of Spacial Redundancy
   - Entropy of a Random Variable
   - Reduction of Entropy
   - Entropy Encoding
3. Extended JP2000 Format
   - Time-Frequency Transform
   - Deadzone Quantization
   - Embedded Block Coding
4. Summary

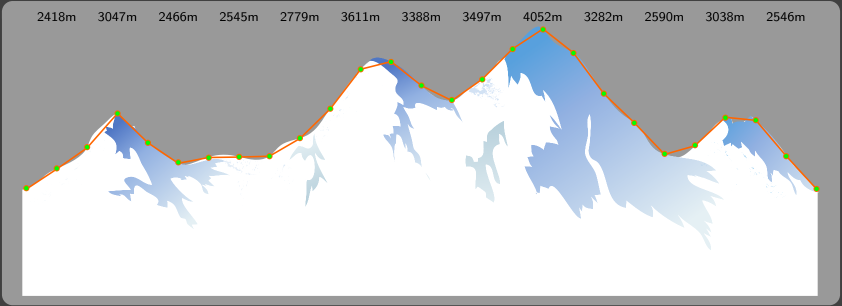
Reduction of Spacial Redundancy

Motivation



Reduction of Spacial Redundancy

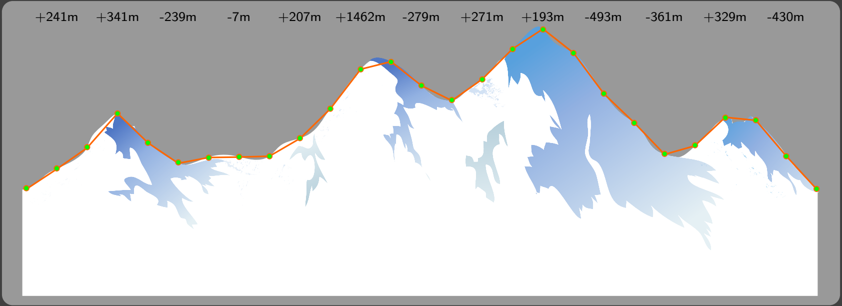
Motivation



$$\sum_{i=0}^{n} b_i^h = 12bit + 12bit + 12bit + 12bit \ldots = 324bit.$$

Reduction of Spacial Redundancy

Motivation

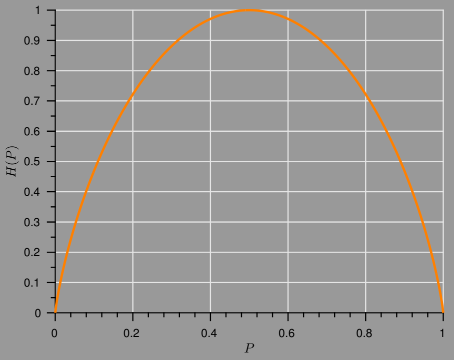


$$\sum_{i=0}^{n} b_i^h = 12bit + 9bit + 10bit + 10bit \ldots = 249bit.$$

Entropy of a Random Variable

$$H(x) = -\sum_{x \in \mathcal{A}_x} f_x(x) \log_2 f_x(x)$$

Entropy of a Random Variable

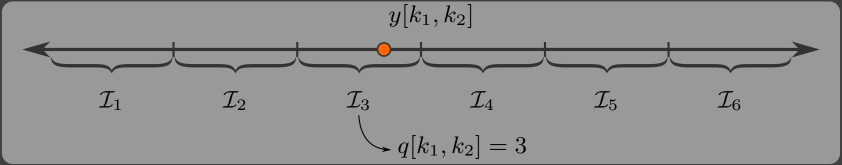


$$X = \begin{cases} 1, & \text{with probability } p, \\ 0, & \text{with probability } 1-p. \end{cases}$$

$$H(X) = -p \log p - (1-p) \log (1-p) := H(P)$$

Reduction of Entropy

Quantization



$$Q\left(y\right) = \begin{cases} i, & \text{if } y[k_1, k2] \in \mathcal{I}_i \\ 0, & \text{otherwise} \end{cases}$$

Entropy Encoding

William Shakespeare

1 Year Old Baby

O, full of scorpions is my mind, dear wife! Thou know'st that Banquo, and his Fleance, lives.

Baa ba babaaababa ba ba ba baba abab babbababa babababaaa baa babababaaaa babaaababa baaabababa

Entropy Encoding

William Shakespeare

1 Year Old Baby

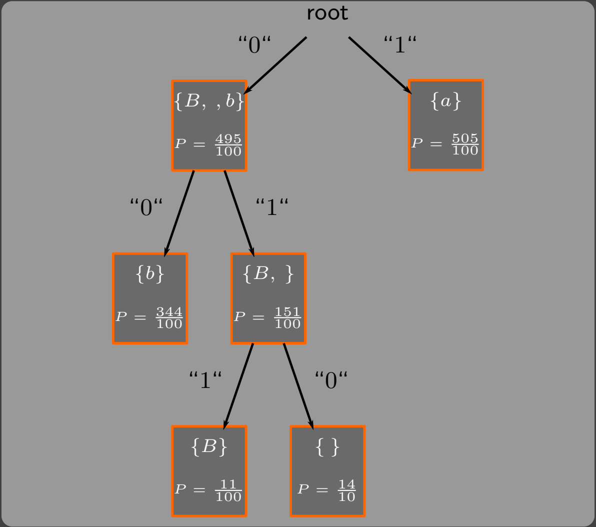
$f()$	$= 0.172;$	$f(!)$	$= 0.011;$	$f(`)$	$= 0.011;$
$f(,)$	$= 0.043;$	$f(.)$	$= 0.011;$	$f(B)$	$= 0.011;$
$f(F)$	$= 0.011;$	$f(O)$	$= 0.011;$	$f(T)$	$= 0.011;$
$f(a)$	$= 0.054;$	$f(c)$	$= 0.022;$	$f(d)$	$= 0.032;$
$\vdots$					

$$H(x) = -\sum_{x \in \mathcal{A}_x} f_x(x) \log_2 f_x(x) = 4.42053$$

$f()$	$= 0.14;$	$f(B)$	$= 0.011;$	$f(a)$	$= 0.505;$
$f(b)$	$= 0.344;$				

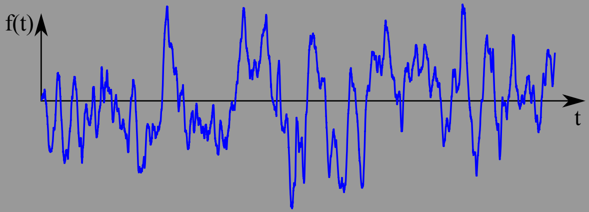
$$H(x) = -\sum_{x \in \mathcal{A}_x} f_x(x) \log_2 f_x(x) = 1.49431$$

Entropy Encoding



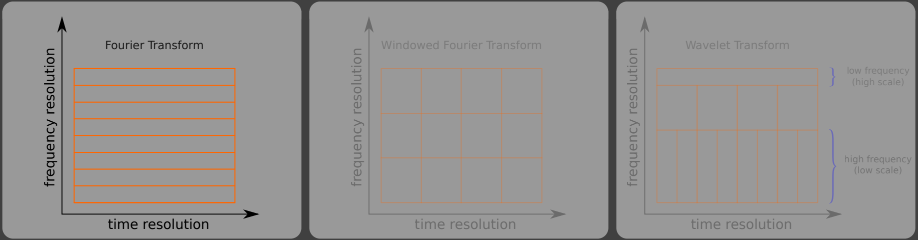
Outline

Time-Frequency Transform



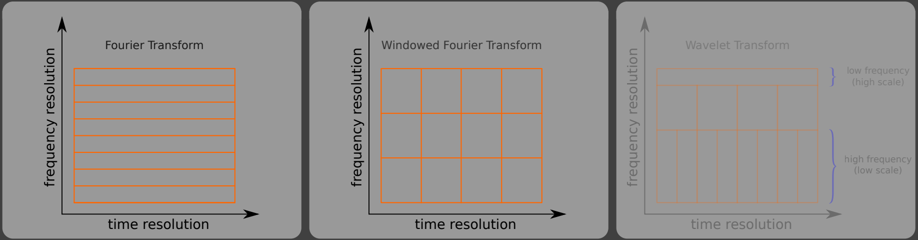
$$\hat{f}\,(\omega) = (2\pi)^{-1/2} \int_{\mathbb{R}} f(x) e^{-i\omega x}\,\mathrm{d}x$$

Time-Frequency Transform



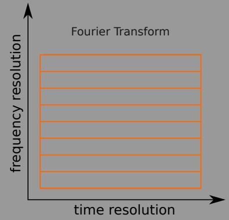
$$\hat{f}\,(\omega) = (2\pi)^{-1/2} \int_{\mathbb{R}} f(x) e^{-i\omega x}\, \mathrm{d}x$$

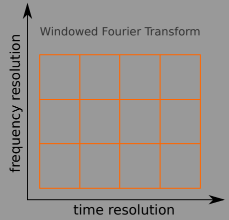
Time-Frequency Transform

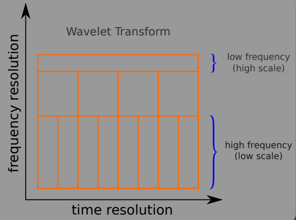


$$G_g f(\omega, t) = (2\pi)^{-1/2} \int_{\mathbb{R}} f(x) g(x - t) e^{-i\omega x} \, \mathrm{d}x$$

Time-Frequency Transform







$$W_\psi f(a,b) = \int_{\mathbb{R}} f(x) \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right) \mathrm{d}x$$

Time-Frequency Transform

Discrete 1D-Wavelet Transform

LeGall-5/3-Wavelet

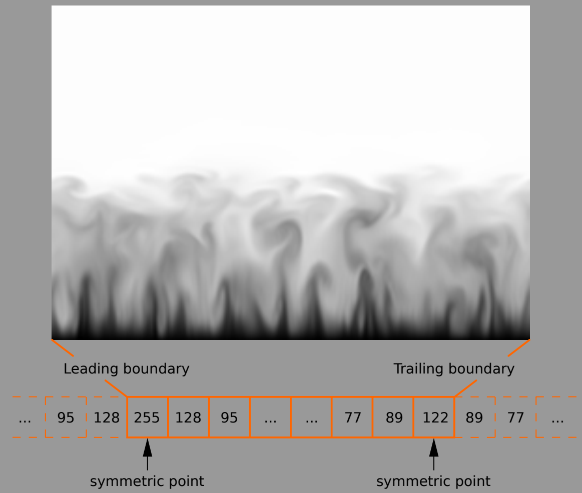
$$\begin{aligned} y(2n+1) &= x(2n+1) - \lfloor \tfrac{x(2n)+x(2n+2)}{2} \rfloor, \\ y(2n) &= x(2n) - \lfloor \tfrac{x(2n-1)+x(2n+1)+2}{4} \rfloor. \end{aligned}$$

Cohen-Daubechies-Feauveau-9/7-Wavelet

$$\begin{aligned} y(2n+1) &\leftarrow x(2n+1) + (\alpha \times |x(2n) + x(2n+2)|), \\ y(2n) &\leftarrow x(2n) + (\beta \times |y(2n-1) + y(2n+1)|), \\ y(2n+1) &\leftarrow x(2n+1) + (\gamma \times |y(2n) + y(2n+2)|), \\ y(2n) &\leftarrow x(2n) + (\delta \times |y(2n-1) + y(2n+1)|), \\ y(2n+1) &\leftarrow -K \times y(2n+1), \\ y(2n) &\leftarrow (1/K) \times y(2n). \end{aligned}$$

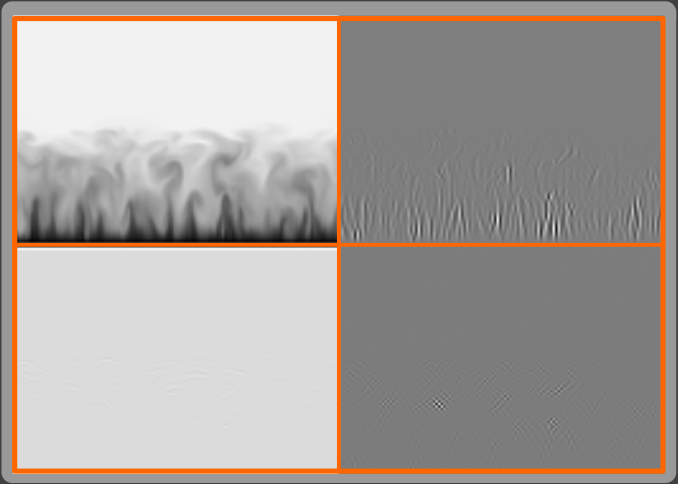
Time-Frequency Transform

Discrete 2D-Wavelet Transform



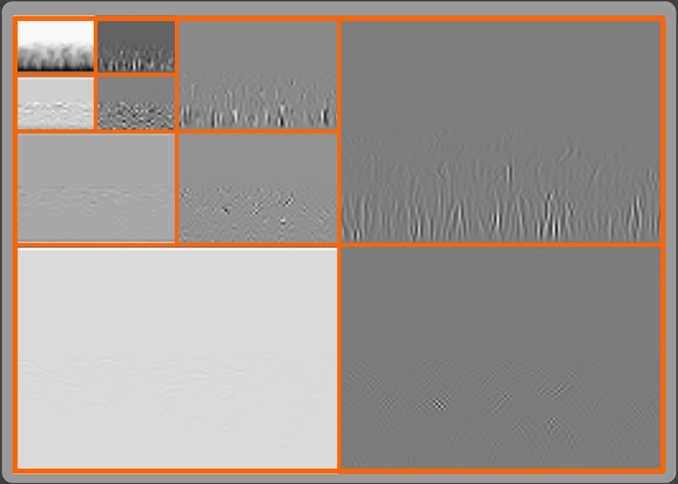
Time-Frequency Transform

Discrete 2D-Wavelet Transform



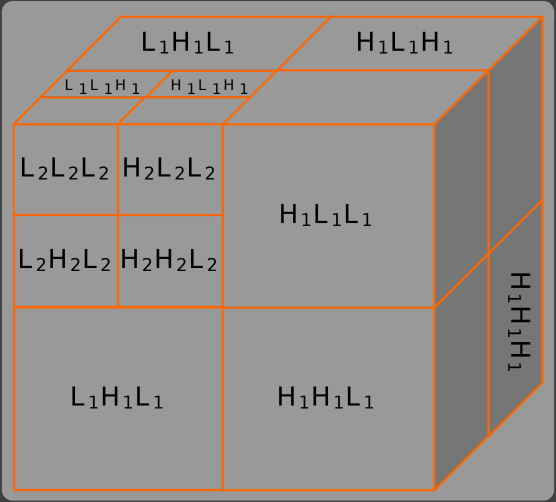
Time-Frequency Transform

Discrete 2D-Wavelet Transform



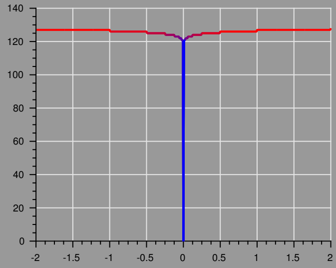
Time-Frequency Transform

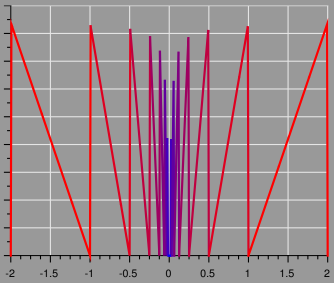
Discrete 3D-Wavelet Transform



Time-Frequency Transform

Floating Point Datasets





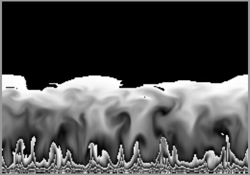
$$(-1)^{sign}\left(1+\sum_{i=1}^{52} b_{52-i}\, 2^{-i}\right)\times 2^{e-1023}$$

Time-Frequency Transform

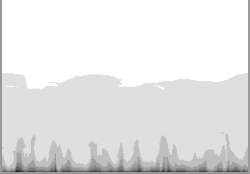
Velocity Field

Sign

Mantissa

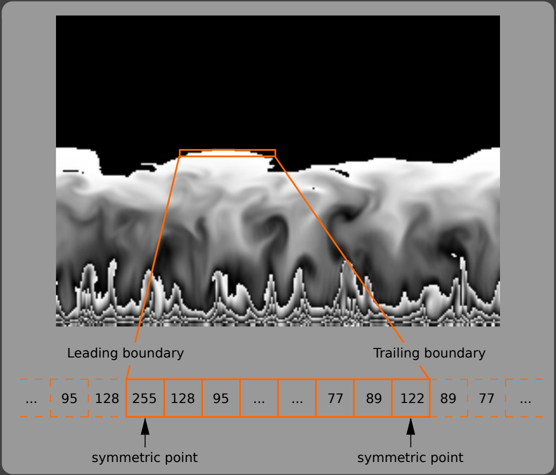


Exponent



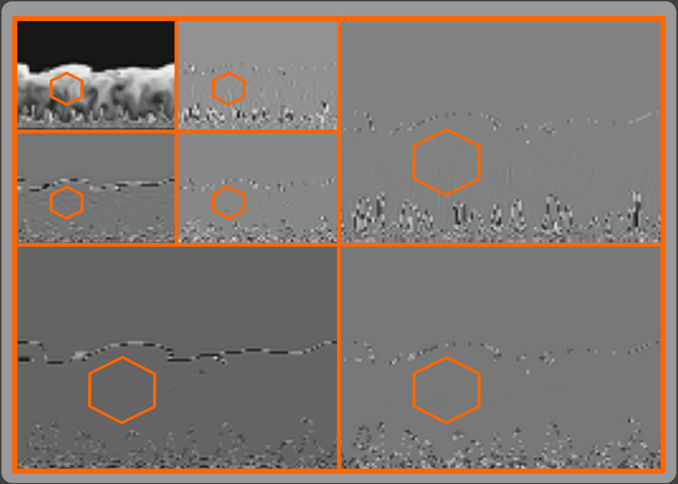
Time-Frequency Transform

Shape Adaptive Wavelet Transform [2]



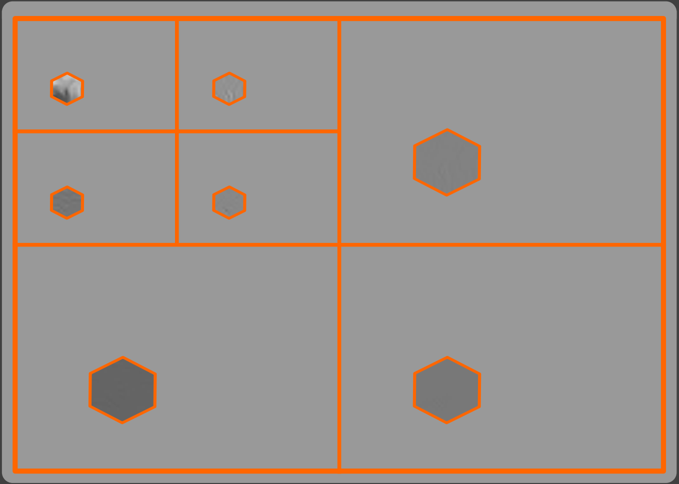
Time-Frequency Transform

Shape Adaptive Wavelet Transform [2]

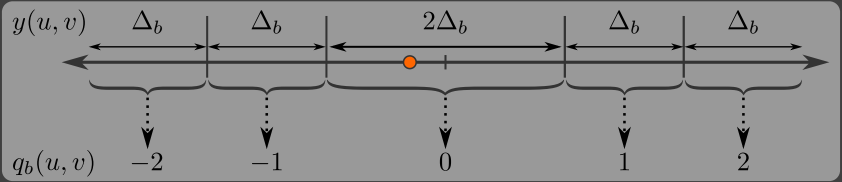


Time-Frequency Transform

Shape Adaptive Wavelet Transform [2]



Deadzone Quantization

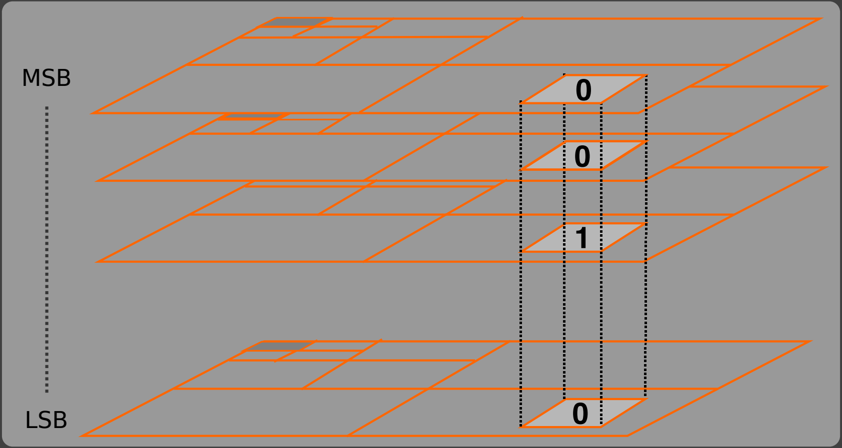


$$\Delta_b = 2^{R_b - \epsilon_b}\left(1 + \frac{\mu_b}{2^{11}}\right),$$

$$q_b(u,v) = \mathrm{sign}(y_b(u,v))\left\lfloor \frac{|y_b(u,v)|}{\Delta_b} \right\rfloor.$$

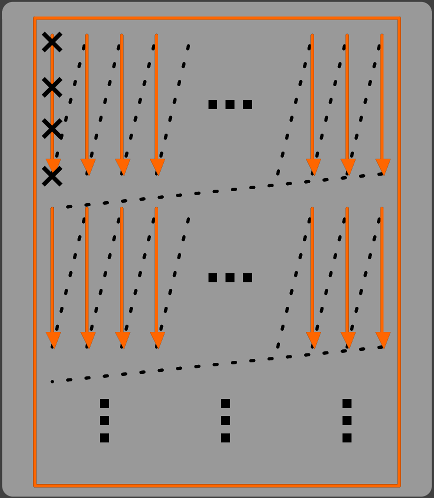
Embedded Block Coding

Coefficient Bit Modeling



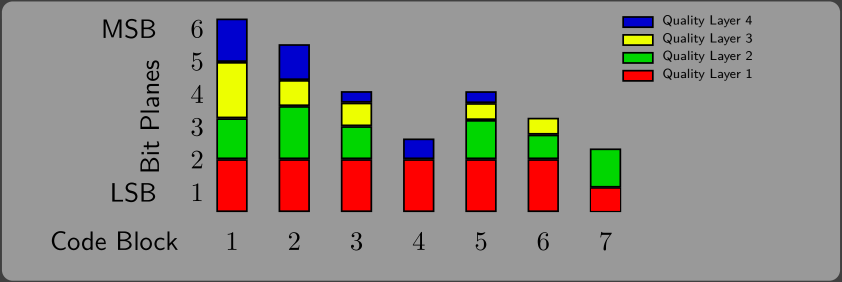
Embedded Block Coding

Coefficient Bit Modeling



Embedded Block Coding

Rate Allocation for IEEE 754 Data [3]



Embedded Block Coding

Rate Allocation for IEEE 754 Data [3]

$$
\begin{aligned}
D &= \textstyle\sum_i G_i \sum_j d_j^i, \\
d_i &= \left(\Delta a_j^i\right)^2 + \left(\Delta b_j^i\right)^2 + \left(\Delta s_j^i\right)^2, \\
\Delta a_j^i &= 2^{23} \cdot \ln(2) \cdot x_j^i \cdot \left\{\Delta y_j^i\right\}_a, \\
\Delta b_j^i &= 2^{23} \cdot s_j^i \cdot 2^{a_j^i} \cdot \left\{\Delta y_j^i\right\}_b, \\
\Delta s_j^i &= 2^{23} \cdot |x_j^i| \cdot \left\{\Delta y_j^i\right\}_s,
\end{aligned}
$$

Outline

Summary

- JPEG 2000 can be extended to support IEEE754 floating point numbers.
- Provides lossless and lossy compression in one codestream.
- Superior compression performance (compared to JPEG).
- Resolution and quality scalability.
- High dynamic range support.
- Robust to bit errors.

Thank you for your attention

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  (visited on 30.03.2016)
- AXIS Communications: Video compression
  URL: http://www.axis.com/de/de/learning/web-articles/technical-guide-to-network-video/video-compression-guide
  (visited on 30.03.2016)