Scalable Multigrid Algorithm for Fluid Dynamic Shape Optimization

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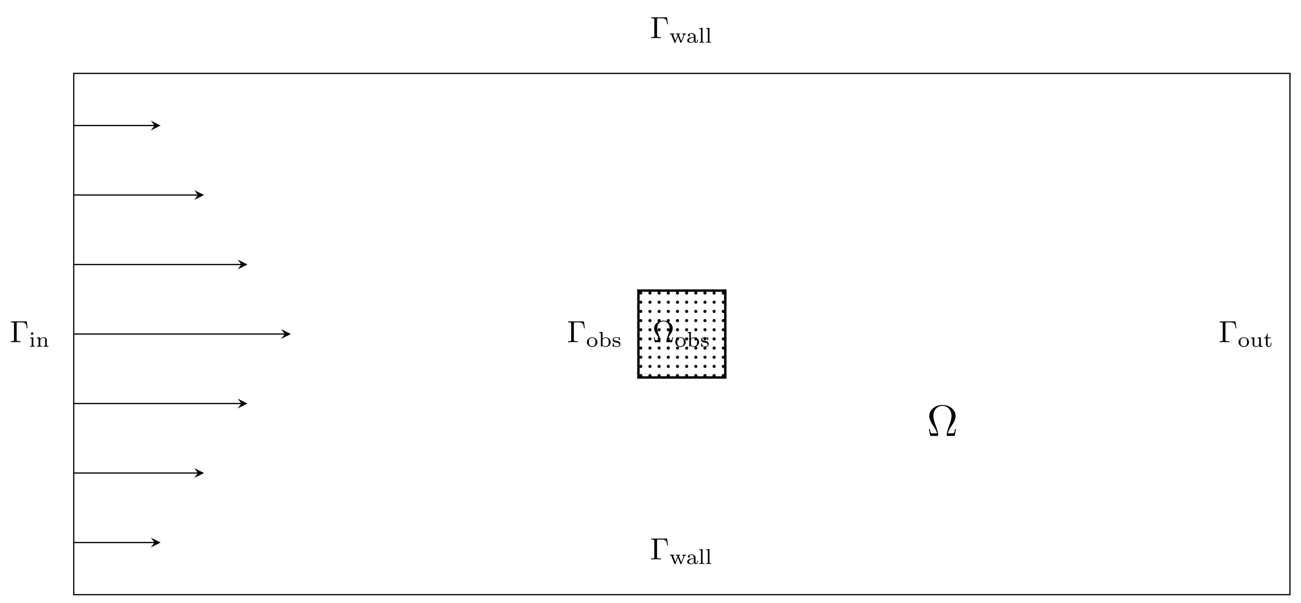
Fluid Dynamic Shape Optimization

- The aim of the project is to further our understanding of optimization schemes for domains that experience large deformations
- Shape optimization is employed to obtain the optimal shape of an obstacle
- This is done in terms of a physical quantity, specifically the **aerodynamic drag**



Our project focuses on the following:

- **Extension operators** that allow for large deformations
- Use of the **method of mappings** to formulate the problem in terms of optimal control [1]
- Mesh quality preservation with respect to the reference configuration, as in [2]

Extension Equations and Method of Mappings

The domain used is sketched above, it consists of:

- An obstacle Ω_{obs} and its surface Γ_{obs} and the holdall domain Ω, which represents a flow tunnel

The algorithm proposed in [3] uses the Augmented Lagrangian. The main blocks are:

- The Navier-Stokes equations for incompressible flow as state equation
- A nonlinear extension equation, solved for the deformation field $\vec{w} \in W$ in terms of η and u

$$\int_\Omega \text{Sym}(D\vec{w}) : D\vec{\delta}_{\vec{w}} + \eta(D\vec{w}\,\vec{w}) \cdot \vec{\delta}_{\vec{w}}\, dx = \int_{\Gamma_{\text{obs}}} u\vec{n} \cdot \vec{\delta}_{\vec{w}}\, ds$$

- The formulation over the reference through the method of mappings, e.g. the cost function is reformulated as

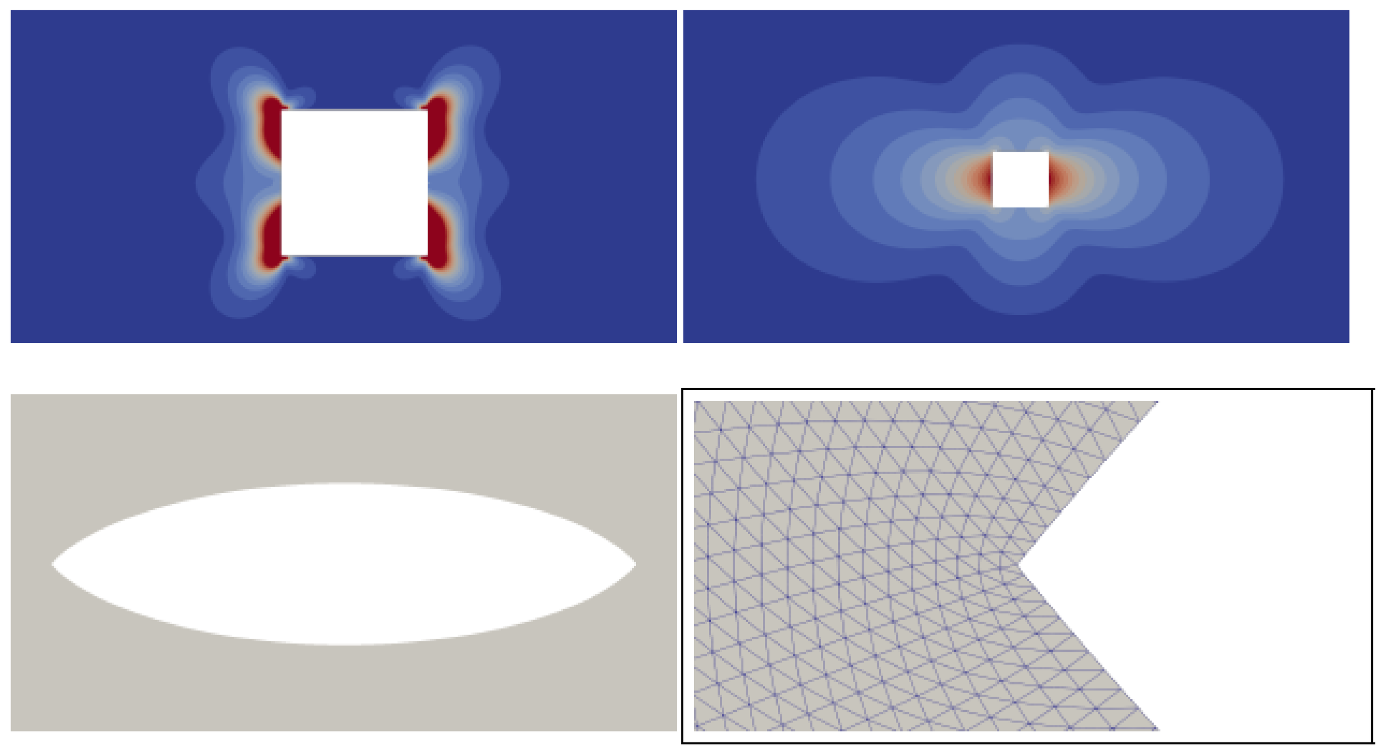
$$j(\vec{v}, \vec{w}) = \nu \int_\Omega (D\vec{v}(DF)^{-1}) : (D\vec{v}(DF)^{-1}) \det(DF) dx.$$

- The volume and barycenter defined by Γ_{obs} are set as constraints

2D and 3D Case Studies

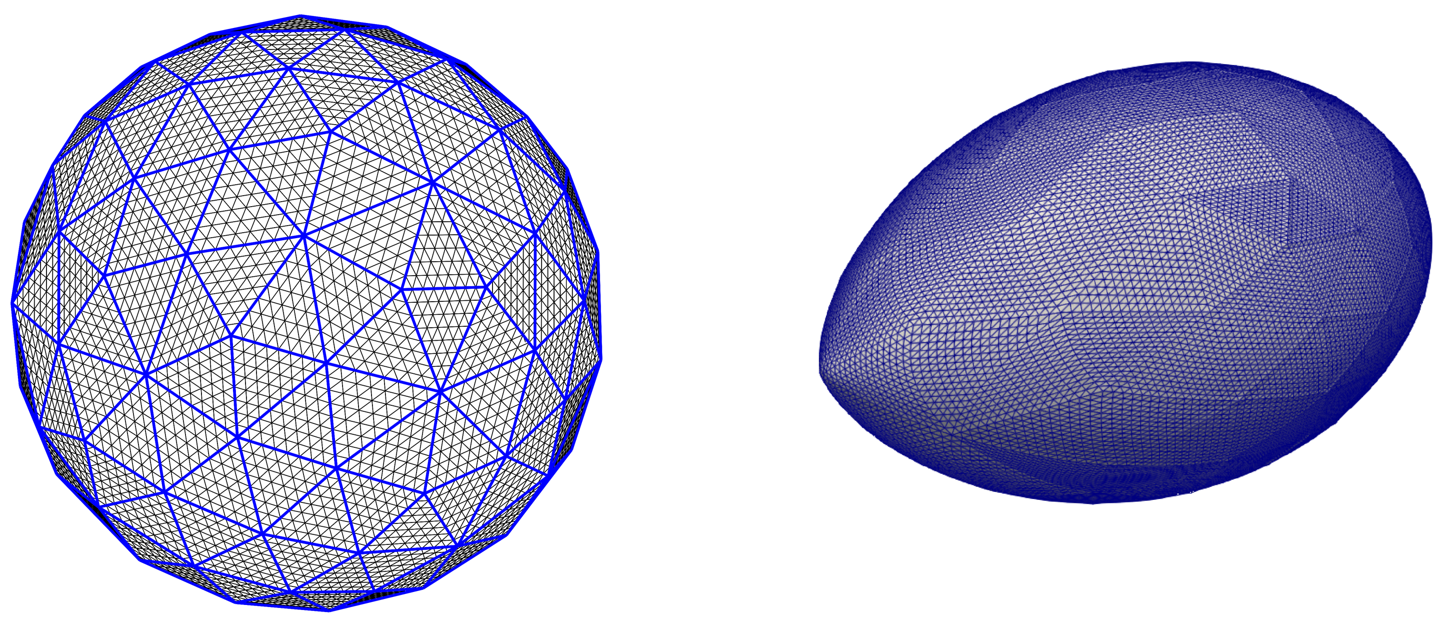
In the 2d results shown below:

- The reference domain included pronounced edges
- In the figure the optimal u, $\vec{w}$ are applied to obtain the optimized Γ_{obs}
- Our algorithm removes and creates surface singularities

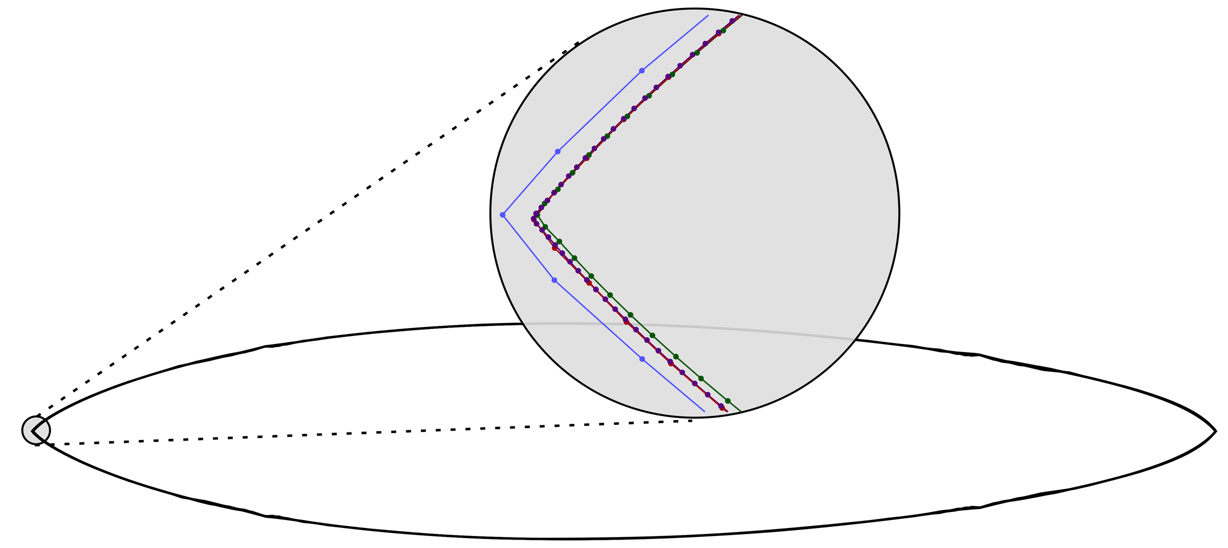


Likewise, for a high viscosity 3d simulation:

- An average of 320 cores were used for each test
- The grid consists of more than 12 million tetrahedrons
- The reference configuration, a rough sphere, is optimized
- Surface singularities are created, as in the 2d case



Grid Independence and Scalability



- Different levels of refinement are applied and compared
- The obtained geometry is the same in all cases
- Superimposed surfaces display minimal differences at the generated tip

Scalability for a massive number of degrees of freedom is featured below with the accumulated iteration counts for weak scaling:

- Increase from 100k triangular elements to more than 6 million
- Almost constant iteration counts for the Newton and linear solvers

Procs	Refs	NumElems	Linear solver (shape derivative)	Newton solver (deformation field)	Linear solver (deformation field)
80	4	105,472	56	12	9
320	5	421,888	70	12	9
1,280	6	1,687,552	77	12	9
5,120	7	6,750,208	77	12	9

Outlook

- A more detailed 3d case study has to be carried out
- The number of cores per node have to be further optimized with respect to the cube topology featured in Hawk
- Novel deformation techniques will be tested as part of the work of the research group for Optimization and Approximation

[1] J. Haubner, M. Siebenborn, and M. Ulbrich, "A continuous perspective on shape optimization via domain transformations," *SIAM Journal on Scientific Computing*, vol. 43, no. 3, A1997–A2018, 2021. DOI: 10.1137/20m1332050.

[2] S. Onyshkevych and M. Siebenborn, "Mesh quality preserving shape optimization using nonlinear extension operators," *Journal of Optimization Theory and Applications*, vol. 16, no. 5, pp. 291–316, Mar. 2021. DOI: 10.1007/s10957-021-01837-8.

[3] J. Pinzon and M. Siebenborn, *Fluid dynamic shape optimization using self-adapting nonlinear extension operators with multigrid preconditioners (in preparation).*

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