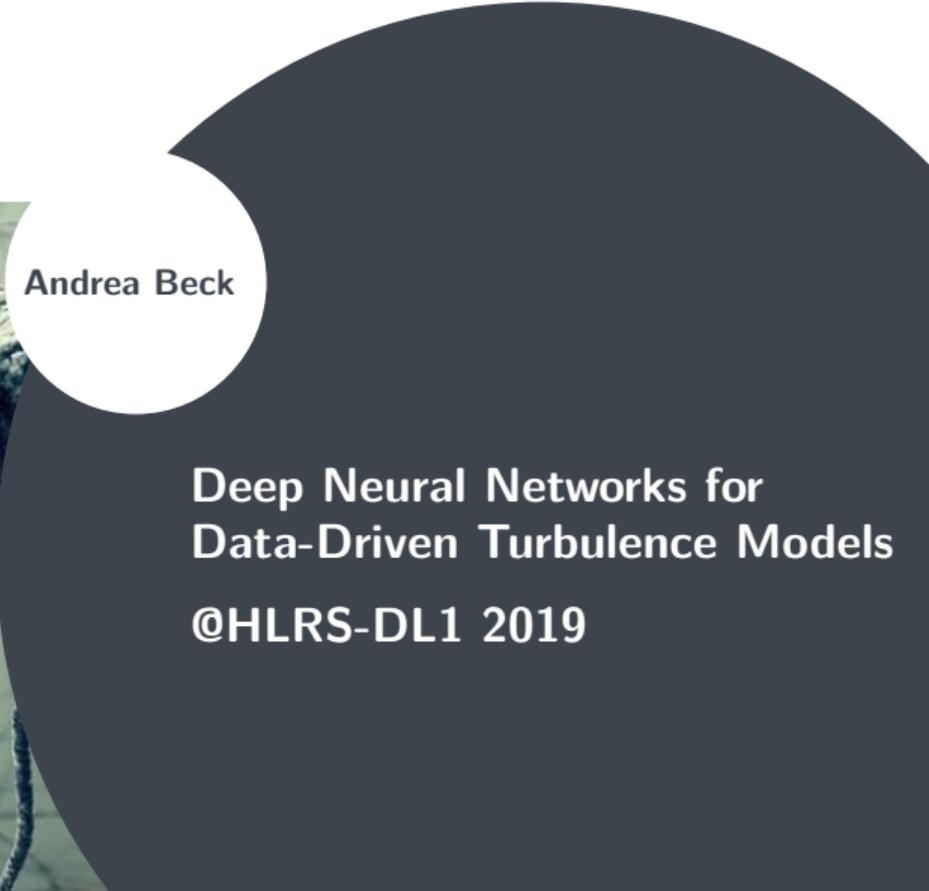


**University of Stuttgart**

Institute of Aerodynamics  
and Gas Dynamics



**Andrea Beck**



**Deep Neural Networks for  
Data-Driven Turbulence Models**

**@HLRS-DL1 2019**

# Outline

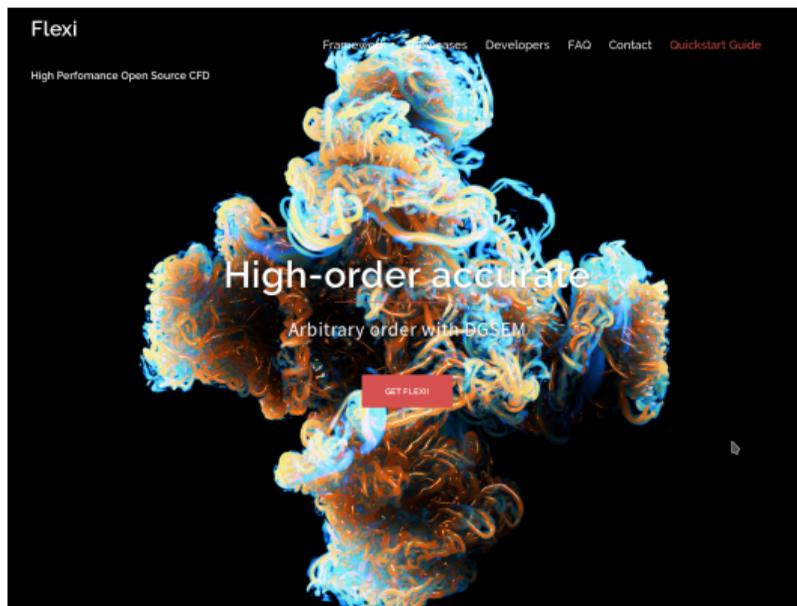
- 1 Introduction
- 2 Machine Learning with Neural Networks
- 3 Turbulence Models from Data
- 4 Training and Results
- 5 Detecting Shocks
- 6 Final Thoughts

# Introduction

1

# Introduction

- Numerics Research Group @ IAG, University of Stuttgart, Germany
- Primary Focus: High Order Discontinuous Galerkin Methods
- OpenSource HPC solver for the compressible Navier-Stokes equations



[www.flexi-project.org](http://www.flexi-project.org)

## DG-SEM in a nutshell

- Hyperbolic/parabolic conservation law , e.g. compressible Navier-Stokes Equations

$$U_t + \vec{\nabla} \cdot \vec{F}(U, \vec{\nabla}U) = 0$$

- Variational formulation and weak DG form per element for the equation system

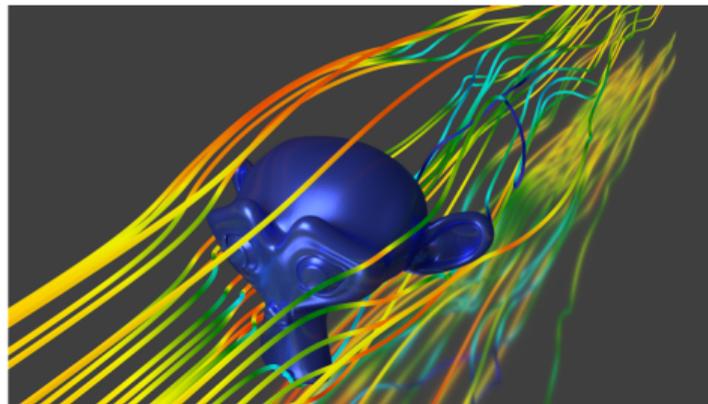
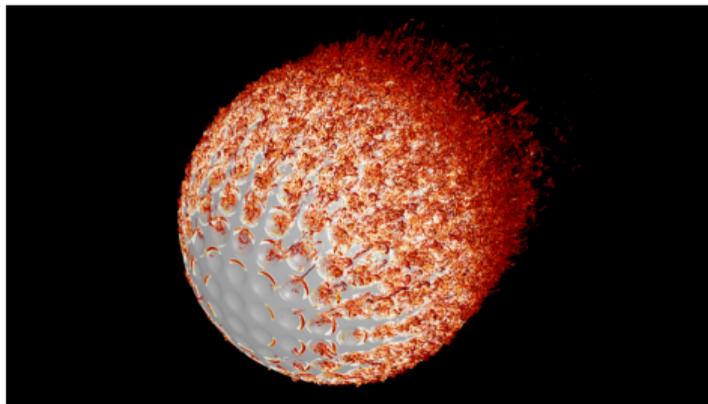
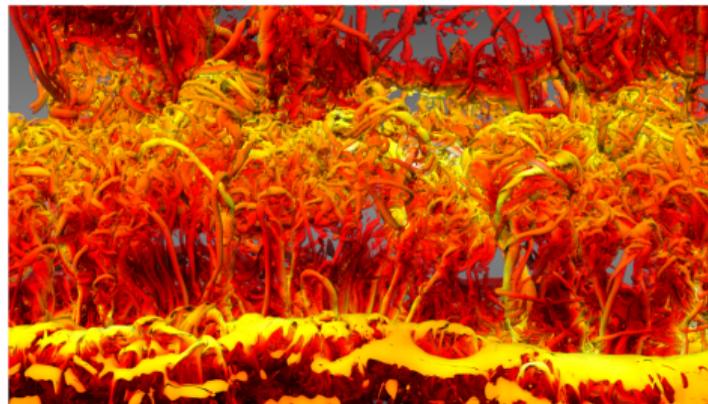
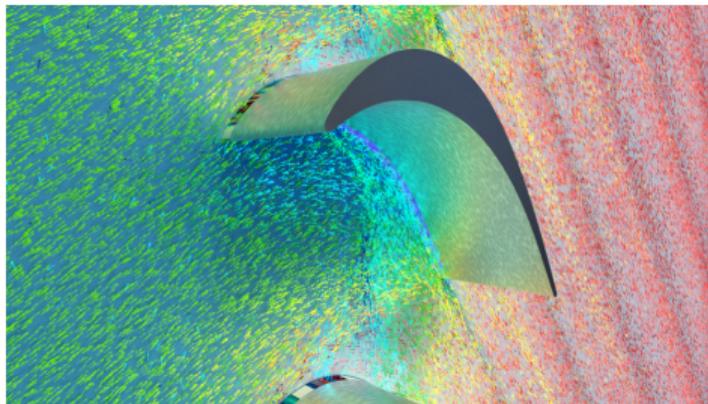
$$\langle JU_t, \psi \rangle_E + \left( \tilde{f}^* \vec{n}_\xi, \psi \right)_{\partial E} - \left\langle \tilde{\vec{F}}, \nabla_\xi \psi \right\rangle_E = 0,$$

- Local tensor-product Lagrange polynomials, interpolation nodes equal to quadrature nodes
- Tensor-product structure in multi-D: line-by-line operations

$$(U_{ij})_t + \frac{1}{J_{ij}} \left[ \tilde{f}^*(1, \eta_j) \hat{\psi}_i(1) - \tilde{f}^*(-1, \eta_j) \hat{\psi}_i(-1) + \sum_{k=0}^N \hat{D}_{ik} \tilde{F}_{kj} \right] \\ + \frac{1}{J_{ij}} \left[ \underbrace{\tilde{g}^*(\xi_i, 1) \hat{\psi}_j(1) - \tilde{g}^*(\xi_i, -1) \hat{\psi}_j(-1) + \sum_{k=0}^N \hat{D}_{jk} \tilde{G}_{ik}}_{\text{1D DGSEM Operator}} \right] = 0$$

- BR1/2 lifting for viscous fluxes, Roe/LF/HLL-type inviscid fluxes, explicit in time by RK/Legendre-Gauss or LGL-nodes

# Applications: LES, moving meshes, acoustics, multiphase, UQ, particle-laden flows...



# Machine Learning with Neural Net- works

2

## Rationale for Machine Learning

“It is very hard to write programs that solve problems like recognizing a three-dimensional object from a novel viewpoint in new lighting conditions in a cluttered scene.

- We don't know what program to write because we **don't know how its done** in our brain.
- Even if we had a good idea about how to do it, the program might be **horrendously complicated.**”

Geoffrey Hinton, computer scientist and cognitive psychologist (h-index:140+)

## Definitions and Concepts

### An attempt at a definition:

**Machine learning** describes algorithms and techniques that progressively improve performance on a specific task through data without being explicitly programmed.

### Learning Concepts

- Unsupervised Learning
- **Supervised Learning**
- Reinforcement Learning

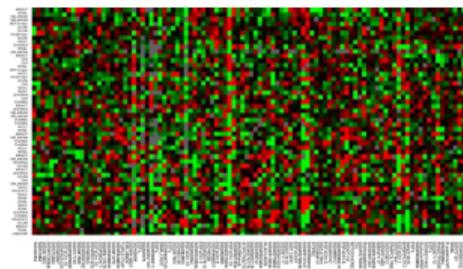
### Artificial Neural Networks

- **General Function Approximators**
- AlphaGo, Self-Driving Cars, Face recognition, NLP
- Incomplete Theory, models difficult to interpret
- NN design: more an art than a science

# Types of ML

## Different Types of Learning:

- Unsupervised learning:  
Discover a good internal representation of the input.  $\Rightarrow$  "Segmentation / Clustering Model"
- Reinforcement learning:  
Learn to select an action to maximize payoff.  $\Rightarrow$  "Behavioral Model"
- Supervised learning:  
Learn to predict an output when given an input vector.  $\Rightarrow$  "Predictive Model"



0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

## History of ANNs

- Some important publications:
  - McCulloch-Pitts (1943): First compute a weighted sum of the inputs from other neurons plus a bias: the perceptron
  - Rosenblatt (1958): First to generate MLP from perceptrons
  - Rosenblatt (1962): Perceptron Convergence Theorem
  - Minsky and Papert (1969): Limitations of perceptrons
  - Rumelhart and Hinton (1986): Backpropagation by gradient descent
  - Cybenko (1989): A NN with a single hidden layer and finite neurons can approximate continuous functions
  - LeCun (1995): “LeNet”, convolutional networks
  - Hinton (2006): Speed-up of backpropagation
  - Krizhevsky (2012): Convolutional networks for image classification
  - Ioffe (2015): Batch normalization
  - He et al. (2016): Residual networks
  - AlphaGo, DeepMind...

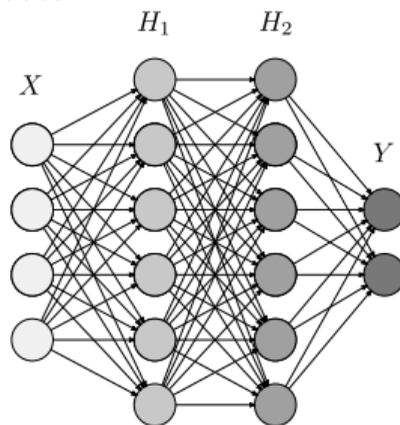
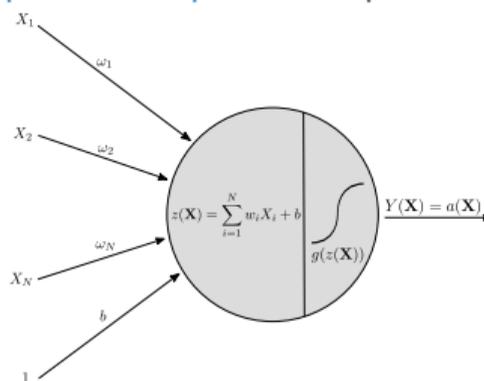
# Neural Networks

- **Artificial Neural Network (ANN):** A non-linear mapping from inputs to outputs:  $\mathbf{M} : \hat{X} \rightarrow \hat{Y}$
- An ANN is a nesting of linear and non-linear functions arranged in a **directed acyclic graph**:

$$\hat{Y} \approx Y = M(\hat{X}) = \sigma_L \left( W_L \left( \sigma_{L-1} \left( W_{L-1} \left( \sigma_{L-2} \left( \dots W_1(\hat{X}) \right) \right) \right) \right) \right), \quad (1)$$

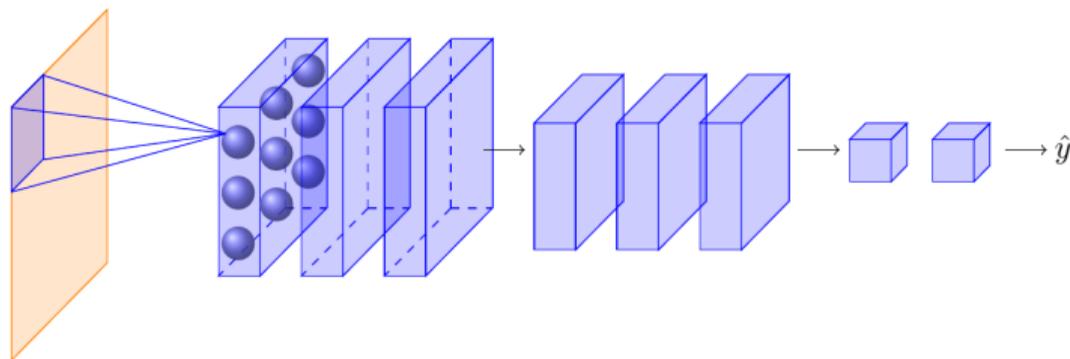
with  $W$  being an affine mapping and  $\sigma$  a non-linear function

- The entries of the mapping matrices  $W$  are the parameters or **weights** of the network: **improved by training**
- Cost function  $C$  as a measure for  $|\hat{Y} - Y|$ , (MSE /  $L_2$  error) convex w.r.t to  $Y$ , but not w.r.t  $W$ :  
 $\Rightarrow$  **non-convex optimization problem** requires a lot of data



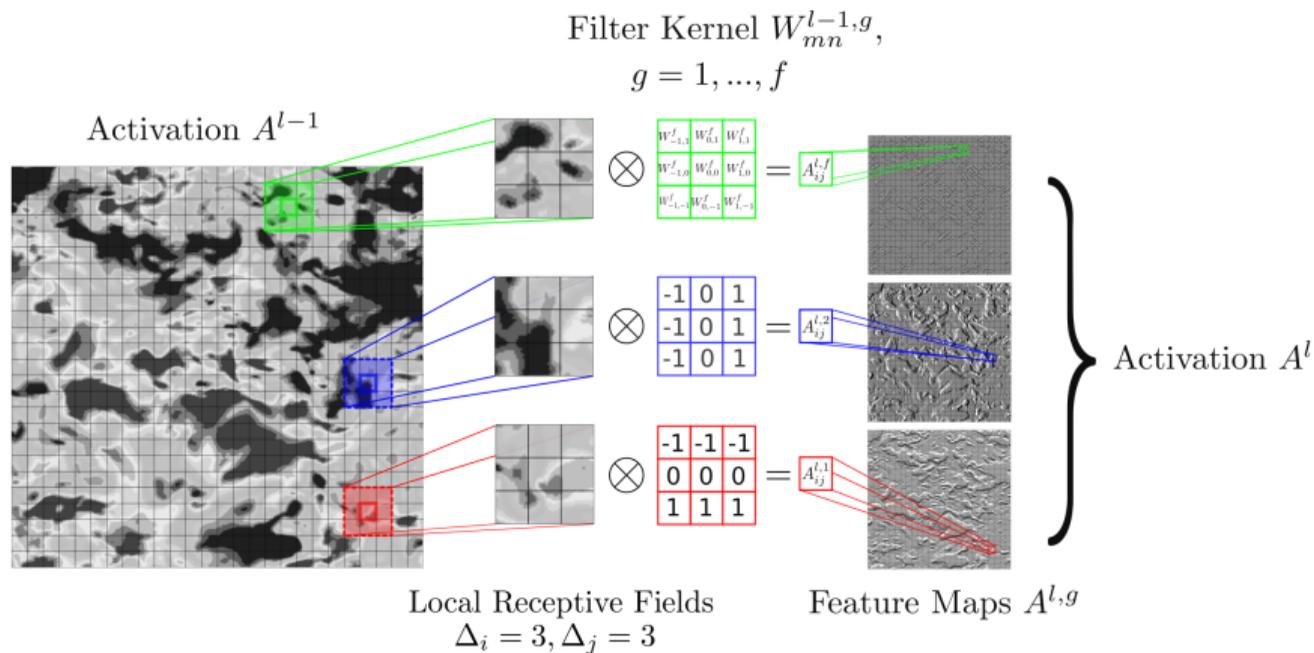
## Advanced Architectures

- Convolutional Neural Networks
  - Local connectivity, **multidimensional trainable filter kernels**, discrete convolution, shift invariance, hierarchical representation
  - Current state of the art for multi-D data and segmentation



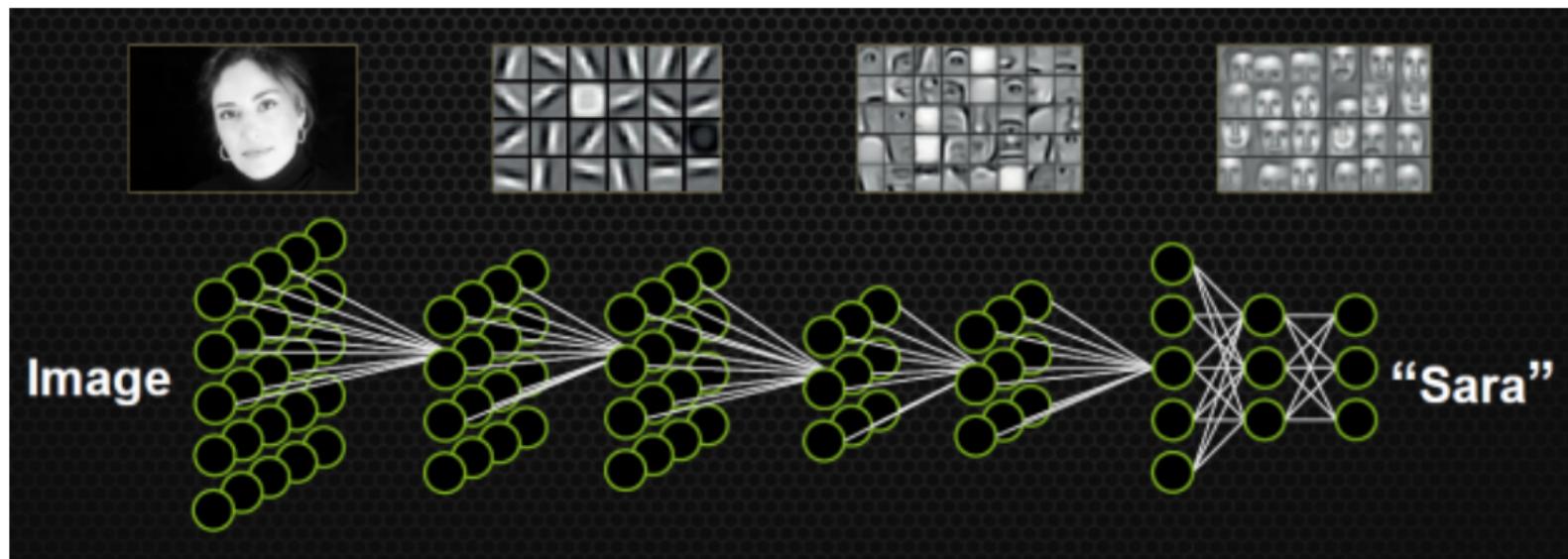
# Advanced Architectures

- Convolutional Neural Networks



## What does a CNN learn?

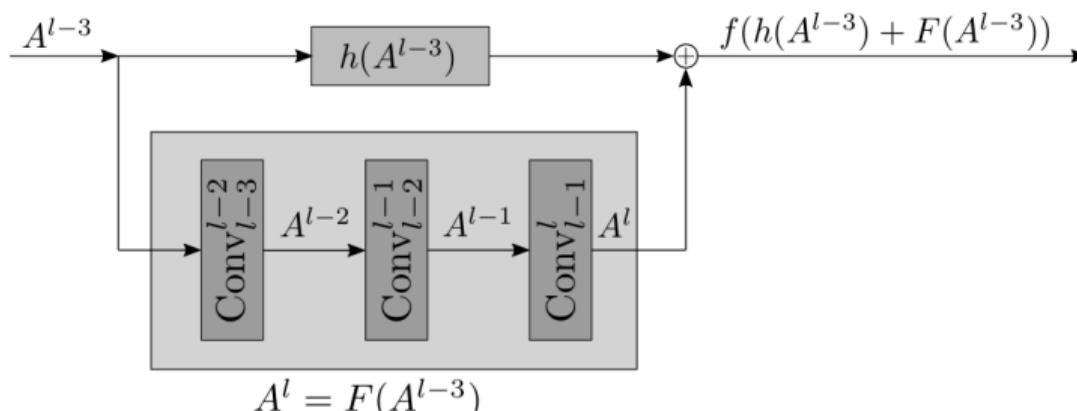
- Representation in hierarchical basis



from: H. Lee, R. Grosse, R. Ranganath, and A. Y. Ng. "Convolutional deep belief networks for scalable unsupervised learning of hierarchical representations." In ICML 2009.

## Residual Neural Networks

- He et al. recognized that the prediction performance of CNNs may deteriorate with depths (not an overfitting problem)
- Introduction of **skip connectors** or **shortcuts**, most often identity mappings
- A sought mapping, e.g.  $G(A^{l-3})$  is split into a **linear** and non-linear (**residual**) part
- Fast passage of the linear part through the network: hundreds of CNN layers possible
- More robust identity mapping



He, Kaiming, et al. "Deep residual learning for image recognition." Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2016.

# Turbulence Models from Data

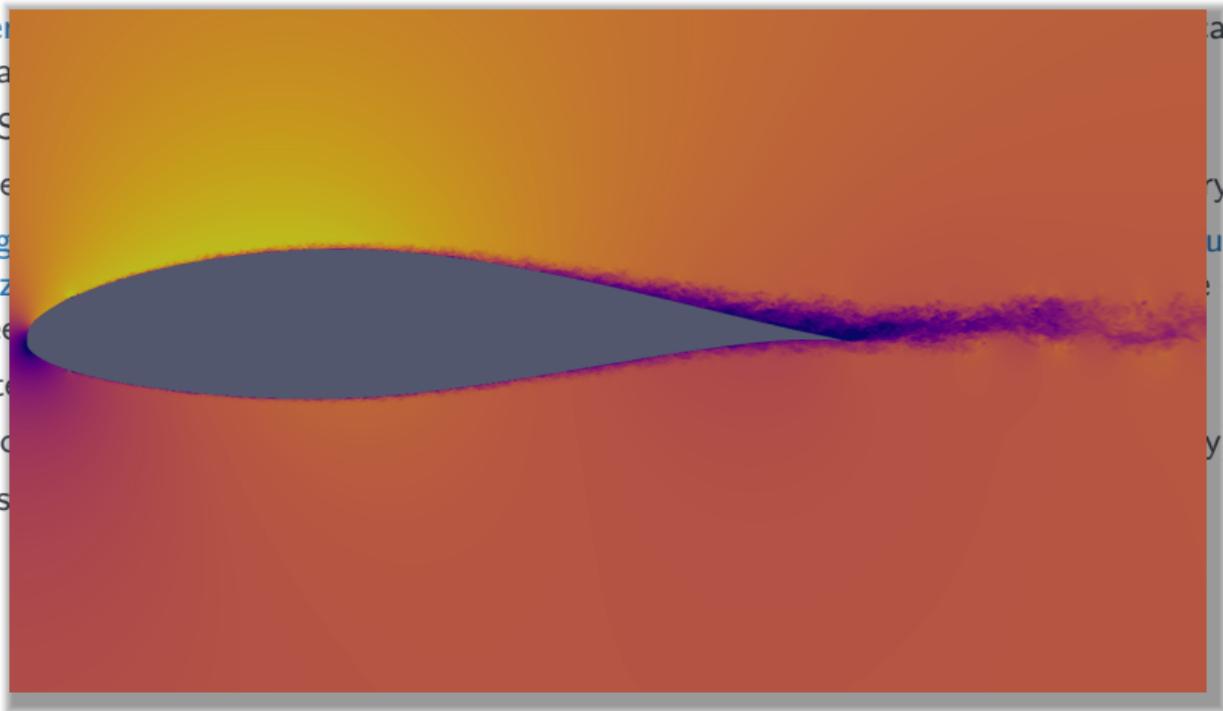
3

## Turbulence in a nutshell

- Turbulent fluid motion is prevalent in naturally occurring flows and engineering applications: multiscale problem in space and time
- Navier-Stokes equations: system of non-linear PDEs (hyp. / parab.)
- Fullscale resolution (DNS) rarely feasible: Coarse scale formulation of NSE is necessary
- Filtering the NSE: Evolution equations for the coarse scale quantities, but with a closure term / regularization dependent on the filtered full scale solution  $\Rightarrow$  Model depending on the coarse scale data needed!
- Two filter concepts: Averaging in time (RANS) or low-pass filter in space (LES)
- An important consequence: RANS can be discretization independent, LES is (typically) not!
- 50 years of research: Still no universal closure model

## Turbulence in a nutshell

- Turbulence is a multiscale phenomenon
- Navier-Stokes equations are ill-posed
- Fullscale simulation is intractable
- Filtering and regularization are essential for data reduction
- Two filter scales are needed
- An important open problem is the derivation of a consistent subgrid-scale model
- 50 years of research



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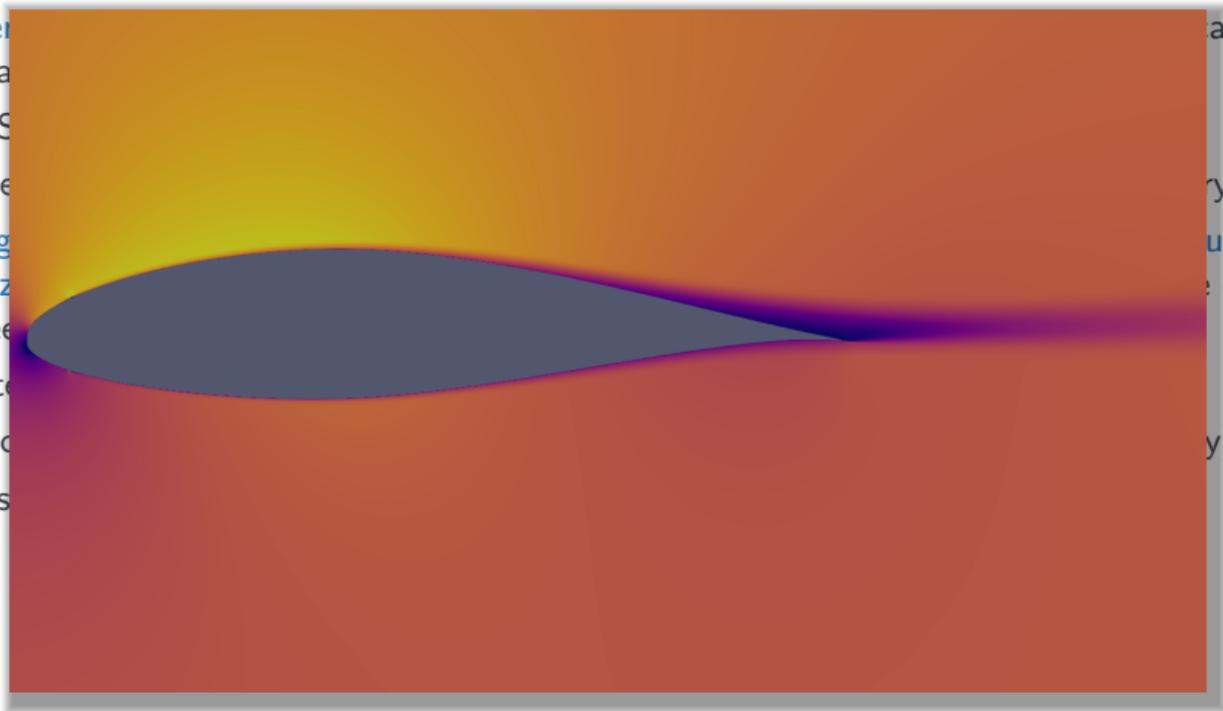
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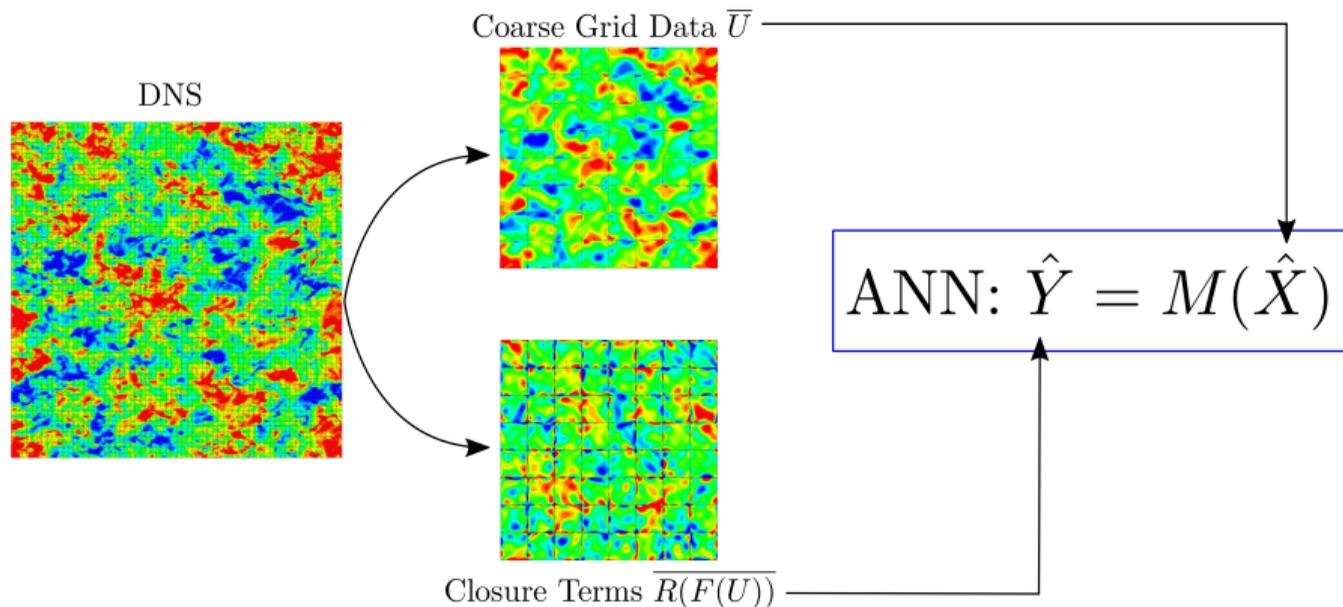
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## Idea

- Approximating an unknown, non-linear and possibly hierarchical mapping from high-dimensional input data to an output  $\Rightarrow$  ANN

## Idea

- Approximating an unknown, non-linear and possibly hierarchical mapping from high-dimensional input data to an output  $\Rightarrow$  LES closure

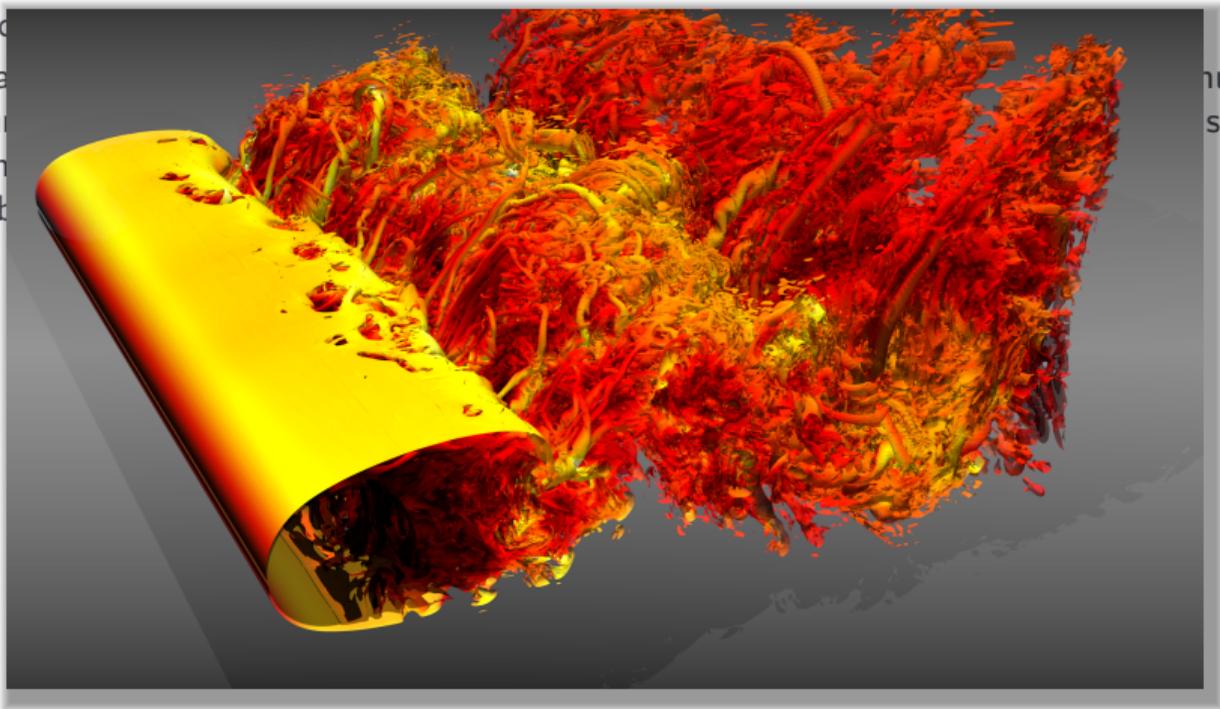


## Problem Definition

- Choice of LES formulations:
  - Scale separation filter: implicit  $\Leftrightarrow$  explicit, linear  $\Leftrightarrow$  non-linear, discrete  $\Leftrightarrow$  continuous...
  - Numerical operator: negligible  $\Leftrightarrow$  part of the LES formulation, isotropic  $\Leftrightarrow$  non-isotropic, commutation with filter...
  - Subgrid closure: implicit  $\Leftrightarrow$  explicit, deconvolution  $\Leftrightarrow$  stochastic modelling,...

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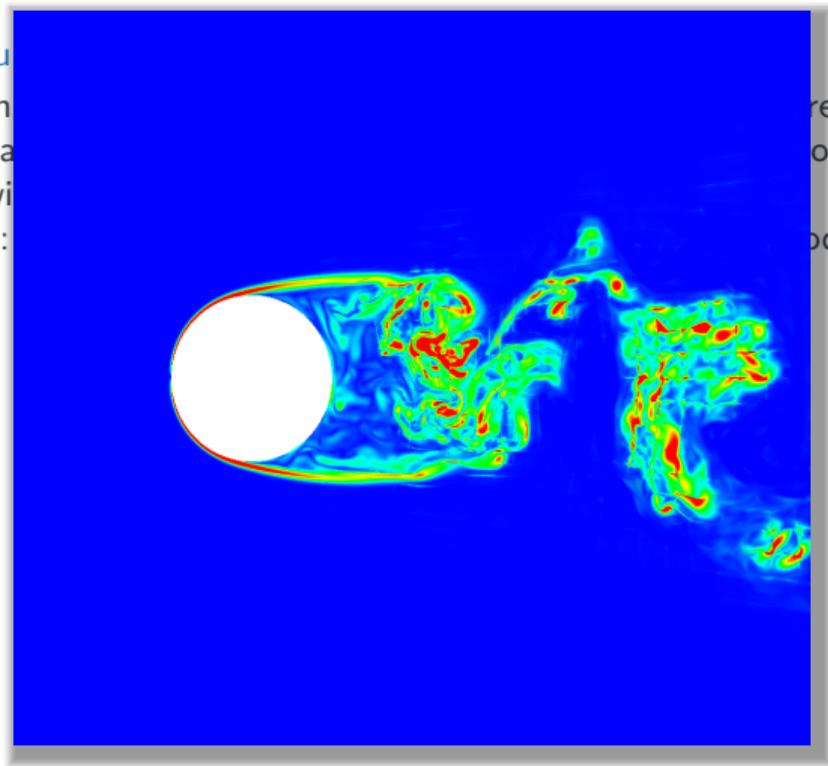
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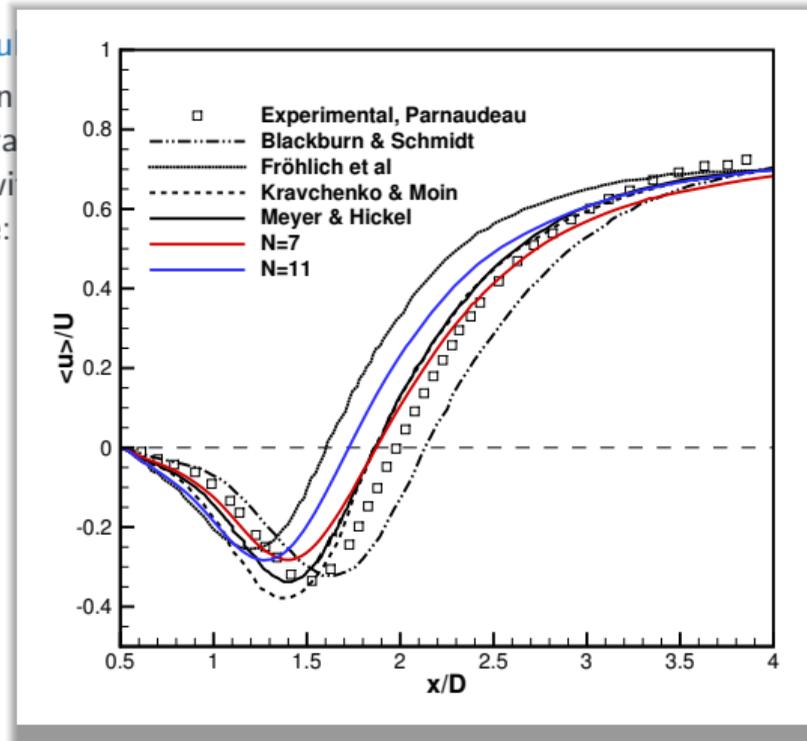
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  - Scale separation
  - Numerical operation commutation with filtering
  - Subgrid closure:



crete  $\Leftrightarrow$  continuous...  
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  - Scale separation
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- Essential for ML methods: Well-defined training data (both input and output)
- Is  $\bar{U}$  known explicitly?  $\Rightarrow$  For practical LES, i.e. grid-dependent LES, it is not most of the time!

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### Definition: Perfect LES

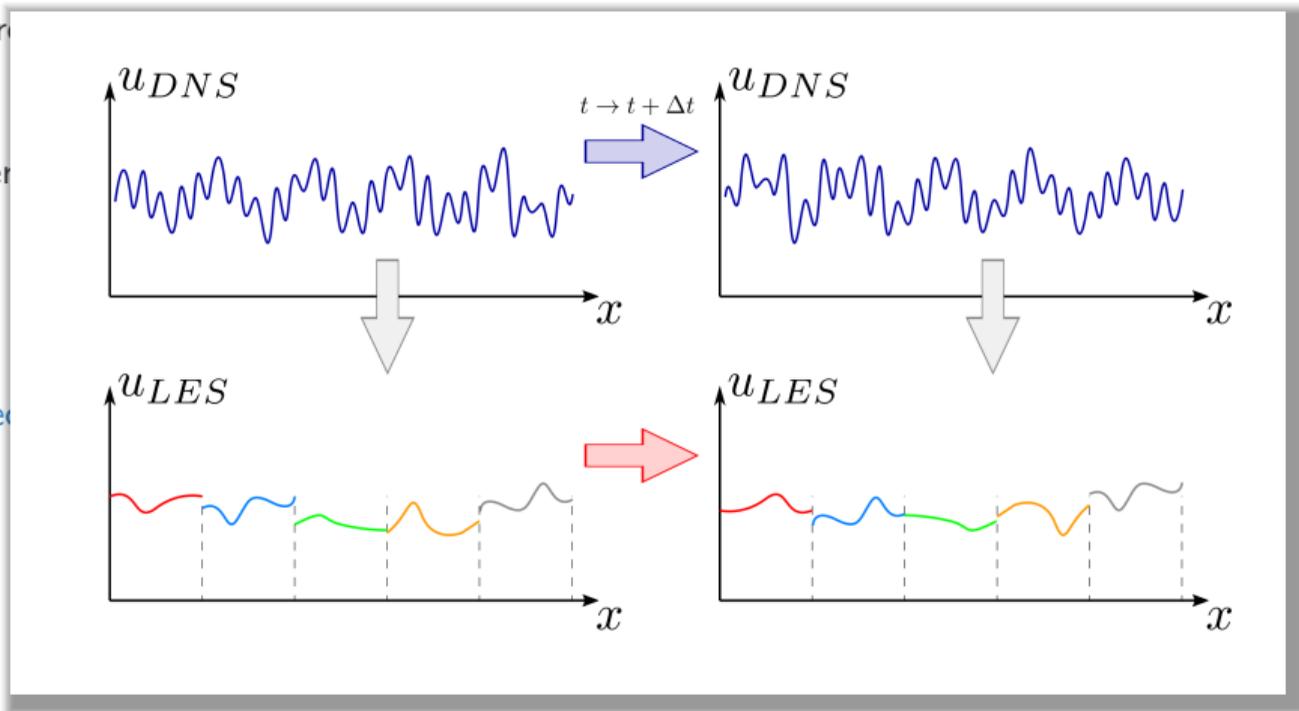
- All terms must be computed on the coarse grid
- Given  $\bar{U}(t_0, x) = \overline{U^{DNS}}(t_0, x) \quad \forall x$ , then  $\bar{U}(t, x) = \overline{U^{DNS}}(t, x) \quad \forall x$  and  $\forall t > 0$

# Turbulence Closure

- Filter

- Imper

- Perfect



(2)

(3)

(4)

## Turbulence Closure

- Filtered NSE:

$$\frac{\partial \bar{U}}{\partial t} + \overline{R(F(U))} = 0 \quad (2)$$

- Imperfect closure with  $\hat{U} \neq \bar{U}$ :

$$\frac{\partial \hat{U}}{\partial t} + \tilde{R}(F(\hat{U})) = \underbrace{\tilde{M}(\hat{U}, C_k)}_{\text{imperfect closure model}}, \quad (3)$$

- Perfect closure with  $\bar{U}$

$$\frac{\partial \bar{U}}{\partial t} + \tilde{R}(F(\bar{U})) = \underbrace{\tilde{R}(F(\bar{U})) - \overline{R(F(U))}}_{\text{perfect closure model}}. \quad (4)$$

- Note  $\tilde{R}(F(\bar{U}))$  is necessarily a part of the closure, but it is **known**
- Perfect LES and perfect closure are not new concepts: introduced by R. Moser et al in a series of papers\*, **termed ideal / optimal** LES

\*Langford, Jacob A. & Robert D. Moser. "Optimal LES formulations for isotropic turbulence." JFM 398 (1999): 321-346.

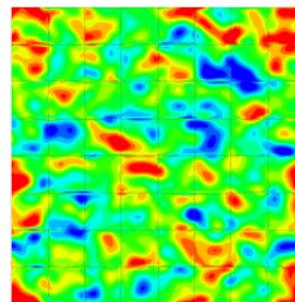
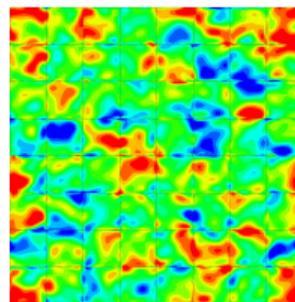
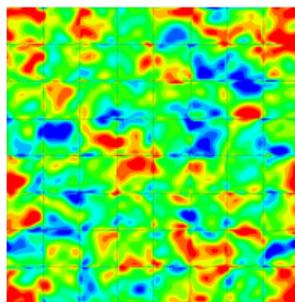
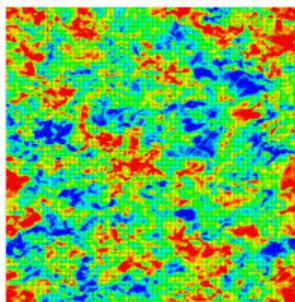
## Perfect LES

$$\frac{\partial \bar{U}}{\partial t} + \overbrace{\tilde{R}(F(\bar{U}))}^{\text{coarse grid operator}} = \underbrace{\overbrace{\tilde{R}(F(\bar{U}))}^{\text{coarse grid operator}} - \overline{R(F(U))}}_{\text{perfect closure model}}.$$

- The specific operator and filter choices are **not relevant** for the perfect LES
- Note that the coarse grid operator is part of the closure (and cancels with the LHS)
- We choose:
  - DNS-to-LES operator  $\bar{(\ )}$ :  $L_2$  projection from DNS grid onto LES grid: We choose a **discrete** scale-separation filter
  - LES operator  $\tilde{(\ )}$ : 6<sup>th</sup> order DG method with split flux formulation and low dissipation Roe flux

## Perfect LES

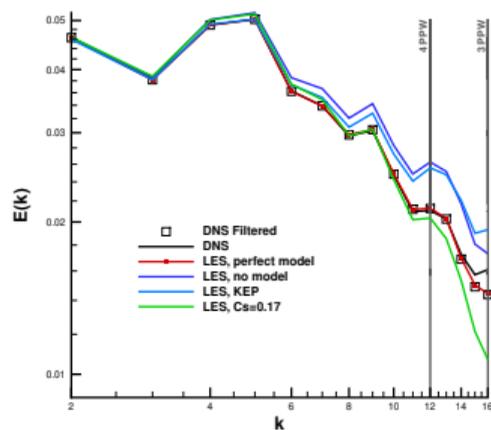
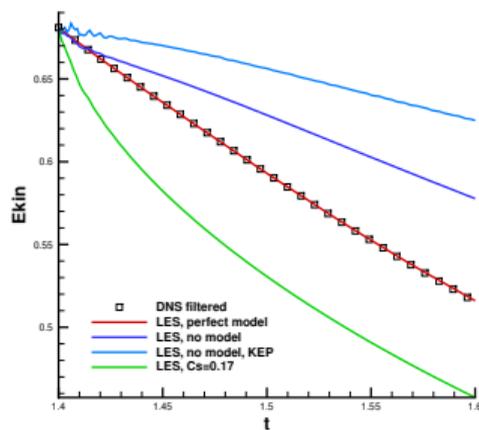
- Perfect LES runs with closure term from DNS
- Decaying homogeneous isotropic turbulence
- DNS grid:  $64^3$  elements,  $N = 7$  : LES grid:  $8^3$  elements.  $N = 5$  :



Left to right: a) DNS, b) filtered DNS, c) computed perfect LES d) LES with Smagorinsky model  
 $C_s = 0.17$

# Perfect LES

- Perfect LES runs with closure term from DNS
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- DNS grid:  $64^3$  elements.  $N = 7$  : LES grid:  $8^3$  elements.  $N = 5$  :



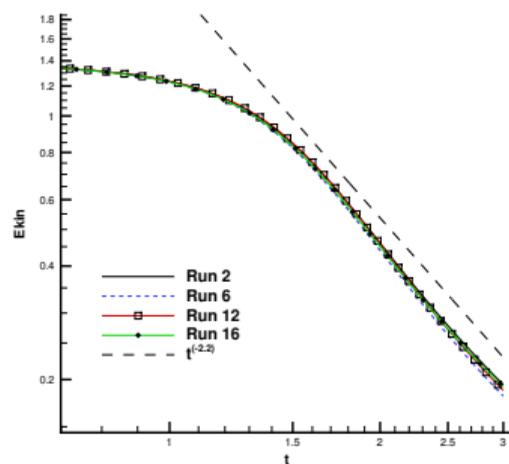
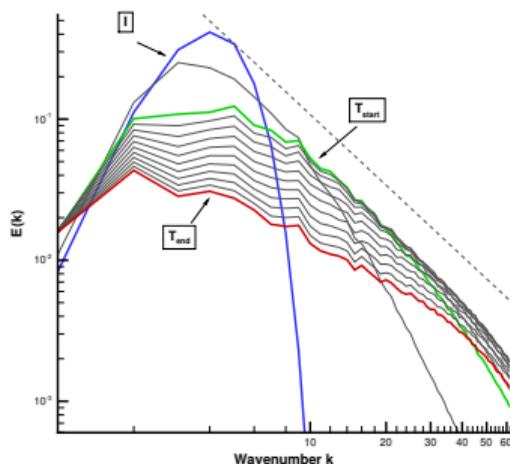
- $\Rightarrow$  Perfect LES gives well-defined target and input data for supervised with NN

# Training and Results

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## Data Acquisition: Decaying Homogeneous Isotropic Turbulence

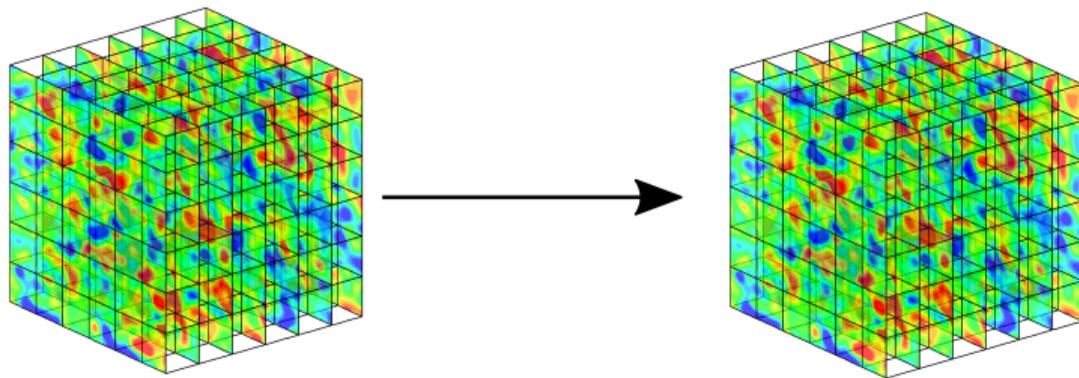
- Ensemble of DNS runs of decaying homogeneous isotropic turbulence with initial spectrum defined by Chasnov (1995) initialized by Rogallo (1981) procedure and  $Re_\lambda = 180$  at start
- Data collection in the range of exponential energy decay: 25 DHIT realizations with 134 Mio DOF each computed on CRAY XC40 (approx. 400,000 CPUh, 8200 cores)
- Compute coarse grid terms from DNS-to-LES operator



## Features and Labels

- Each sample: A single LES grid cell with  $6^3$  solution points
- Input features: velocities and LES operator:  $\overline{u}_i, \tilde{R}(F(\overline{U}))$
- Output labels: DNS closure terms on the LES grid  $\overline{R(F(U))}$

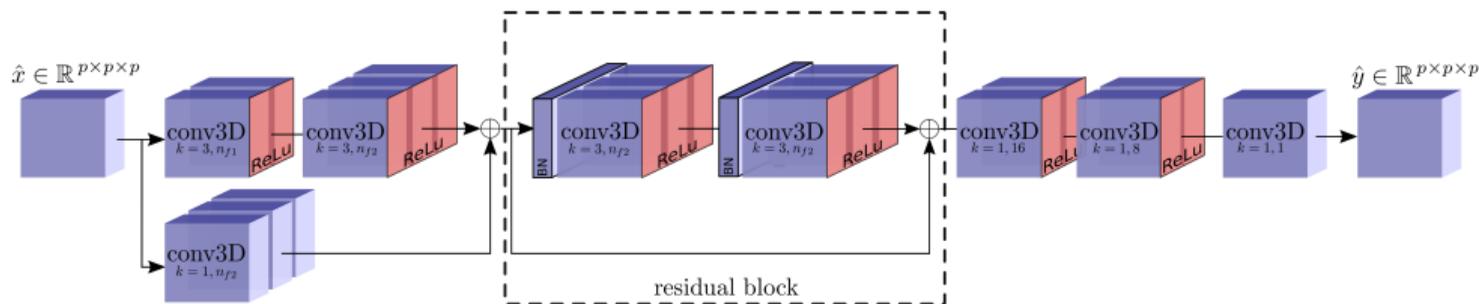
$$\hat{X} = \left\{ \hat{x} \in \mathbb{R}^{6 \times p \times p \times p} \mid \hat{x} = (\overline{u}_{ijk}, \overline{v}_{ijk}, \overline{w}_{ijk}, \tilde{R}(F(\overline{U}^1))_{ijk}, \tilde{R}(F(\overline{U}^2))_{ijk}, \tilde{R}(F(\overline{U}^3))_{ijk}), \text{ with } i, j, k = 0, \dots, p-1 \right\}$$



$$\hat{Y} = \left\{ \hat{y} \in \mathbb{R}^{3 \times p \times p \times p} \mid \hat{y} = \overline{R(F(U))}_{ijk}^n, \text{ with } n = 1, \dots, 3; i, j, k = 0, \dots, p-1 \right\}$$

# Networks and Training

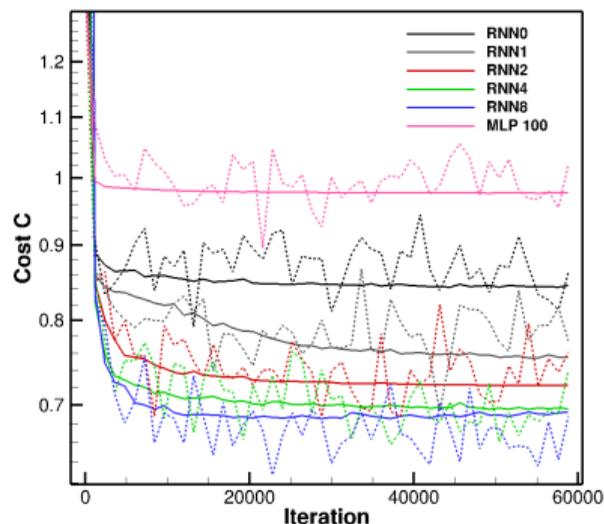
- CNNs with **skip connections** (RNN), batch normalization, ADAM optimizer, data augmentation
- Different **network depths** (no. of residual blocks)
- For comparison: MLP with 100 neurons in 1 hidden layer\*
- Implementation in Python / Tensorflow, Training on K40c and P100 at HLRS
- Split in training, semi-blind validation and blind test DHIT runs



\*Gamahara & Hattori. "Searching for turbulence models by artificial neural network." Physical Review Fluids 2.5 (2017)

## Training Results I: Costs

- Cost function for different network depths
- RNNs outperform MLP, **deeper networks learn better**
- The approach is **data-limited!** NNs are very data-hungry!



## Training Results II: Correlation

Network	$a, b$	$\mathcal{CC}(a, b)$	$\mathcal{CC}^{inner}(a, b)$	$\mathcal{CC}^{surf}(a, b)$
RNN0	$\overline{R(F(U))^1}, \overline{R(F(U))^1}^{ANN}$	0.347676	0.712184	0.149090
	$\overline{R(F(U))^2}, \overline{R(F(U))^2}^{ANN}$	0.319793	0.663664	0.134267
	$\overline{R(F(U))^3}, \overline{R(F(U))^3}^{ANN}$	0.326906	0.669931	0.101801
RNN4	$\overline{R(F(U))^1}, \overline{R(F(U))^1}^{ANN}$	0.470610	0.766688	0.253925
	$\overline{R(F(U))^2}, \overline{R(F(U))^2}^{ANN}$	0.450476	0.729371	0.337032
	$\overline{R(F(U))^3}, \overline{R(F(U))^3}^{ANN}$	0.449879	0.730491	0.269407

- High correlation achievable with deep networks
- For surfaces: one-sidedness of data / filter kernels

## Training Results III: Feature Sensitivity

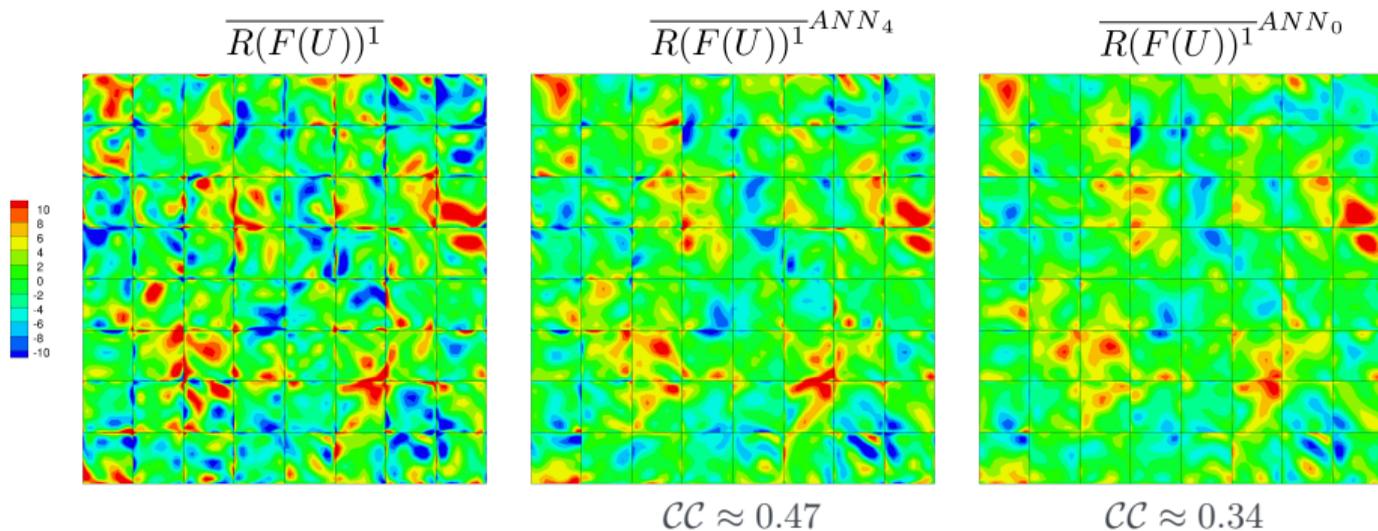
Set	Features	$\mathcal{CC}^1$	$\mathcal{CC}^2$	$\mathcal{CC}^3$
1	$u_i, \tilde{R}(F(\overline{U}^i)), i = 1, 2, 3$	0.4706	0.4505	0.4499
2	$u_i, i = 1, 2, 3$	0.3665	0.3825	0.3840
3	$\tilde{R}(F(\overline{U}^i)), i = 1, 2, 3$	0.3358	0.3066	0.3031
4	$\rho, p, e, u_i, \tilde{R}(F(\overline{U}^i)), i = 1, 2, 3$	0.4764	0.4609	0.4580
5	$u_1, \tilde{R}(F(\overline{U}^1))$	0.3913		

Feature sets and resulting test correlations.  $\mathcal{CC}^i$  with  $i = 1, 2, 3$  denotes the cross correlation between the targets and network outputs  $\mathcal{CC}(\overline{R(F(U)^i)}, \overline{R(F(U))^i}^{ANN})$ . Set 1 corresponds to the original feature choice; Set 5 corresponds to the RNN4 architecture, but with features and labels for the  $u$ -momentum component only.

- Both the coarse grid primitive quantities as well as the **coarse grid operator** contribute strongly to the learning success
- Better learning for 3D cell data than pointwise data

## Training Results IV: Visualization

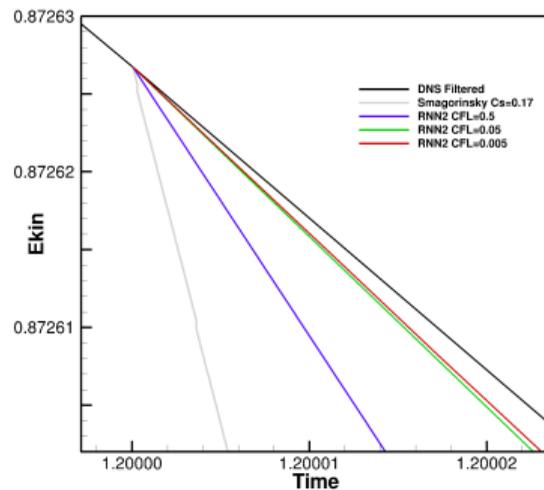
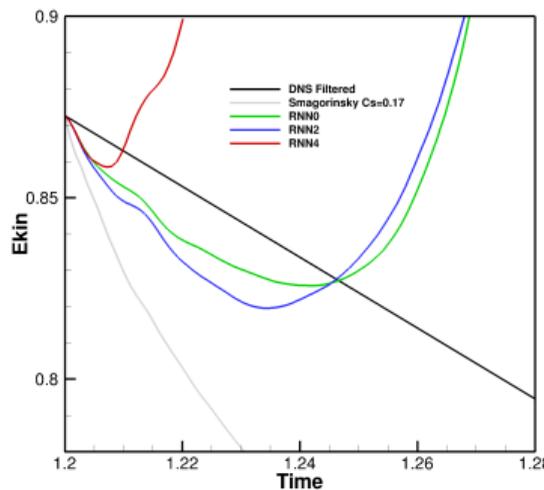
- "Blind" application of the trained network to unknown test data
- Cut-off filter: no filter inversion / approximate deconvolution



## LES with NN-trained model I

$$\frac{\partial \bar{U}}{\partial t} + \tilde{R}(F(\bar{U})) = \tilde{R}(F(\bar{U})) - \underbrace{\overline{R(F(U))}}_{\text{ANN closure}}.$$

- Perfect LES is possible, but the NN-learned mappings are approximate
- No long term stability, but short term stability and dissipation

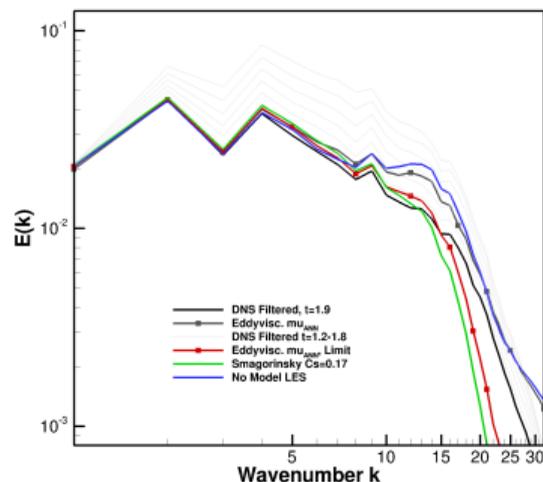
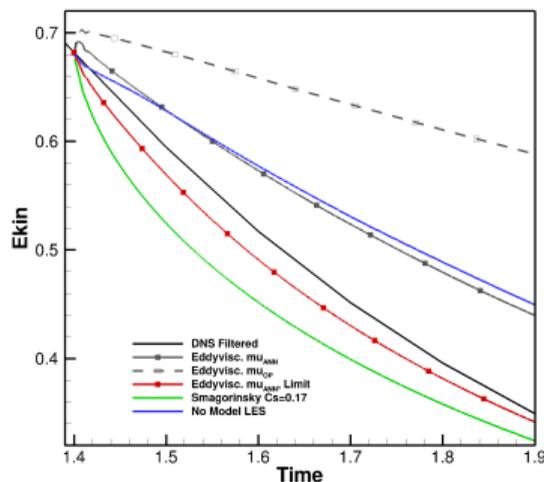


## LES with NN-trained model II

$$\frac{\partial \bar{U}}{\partial t} + \tilde{R}(F(\bar{U})) = \underbrace{\tilde{R}(F(\bar{U})) - \overline{R(F(U))}}_{\text{data-based eddy viscosity model}}.$$

- Simplest model: Eddy viscosity approach with  $\mu_{ANN}$  from

$$\tilde{R}(F(\bar{U}^i)) - \overline{R(F(U^i))} \approx \mu_{ANN} \tilde{R}(F^{visc}(\bar{U}^i, \nabla \bar{U}^i)) \quad (5)$$



## Summary

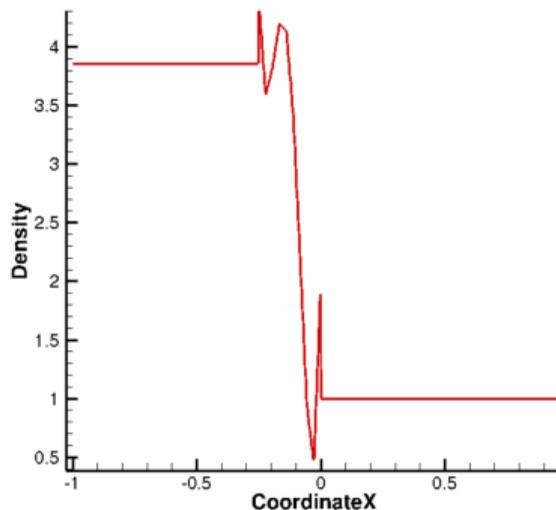
- Perfect / optimal LES framework: well-defined target quantities for learning
- Learning the exact closure terms from data is possible
- Deeper RNNs learn better
- High order methods are a natural fit to CNN: volume data
- Our process is data-limited, i.e. learning can be improved with more data
- Achievable  $CC \approx 45\%$ , with up to  $\approx 75\%$  for inner points
- Both the coarse grid velocities and the coarse grid operator contribute strongly to learning
- The resulting ANN models are dissipative
- No long term stability due to approximate model
- Simplest way to construct a stable model: Data-informed, local eddy-viscosity
- Other approaches to construct models from prediction of closure terms under investigation

# Detecting Shocks

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## Shock Localization through Holistic Edge Detection

- Another quick example of combining CFD + ML
- Shocks and sharp discontinuities cause Gibb's oscillations in high order methods due to **non-smoothness**
- These features need to be treated with special numerical methods to ensure **stability**

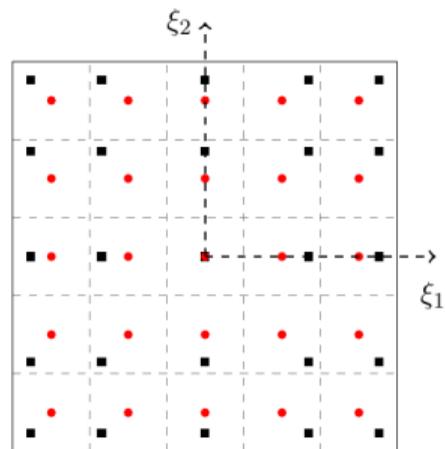


## Shock capturing

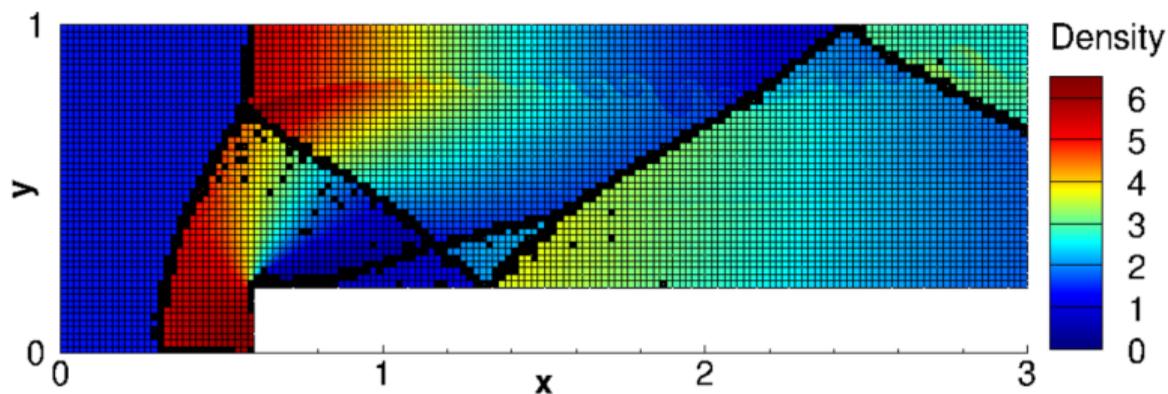
- A **classical** approach:
  1. Choose some numerical method for the stable approximation of discontinuities (e.g. FV subcells, p-reduction, artificial viscosity)
  2. Define a "troubled cell" indicator with empirical parameters
  3. Apply the method from (1) in the troubled cells
  4. Find "good" parameters for (2), where good means both stable and as sharp as possible
  5. Rinse and repeat for different physics, numerics, etc.
- Note that the indicator and the numerics are **closely linked**
- An indicator that leads to a stable simulation for one case (e.g. for one Riemann solver, N, Mach number) will fail for another case
- The troubled cell indicator is an empirically tuned "**tolerance level**" fitted to the numerical scheme: How strong can the discontinuity be for the scheme to survive?
  - ⇒ Shock capturing and shock detection are interdependent
  - ⇒ Experience / Parameter Tuning required

## A DG method for shock capturing

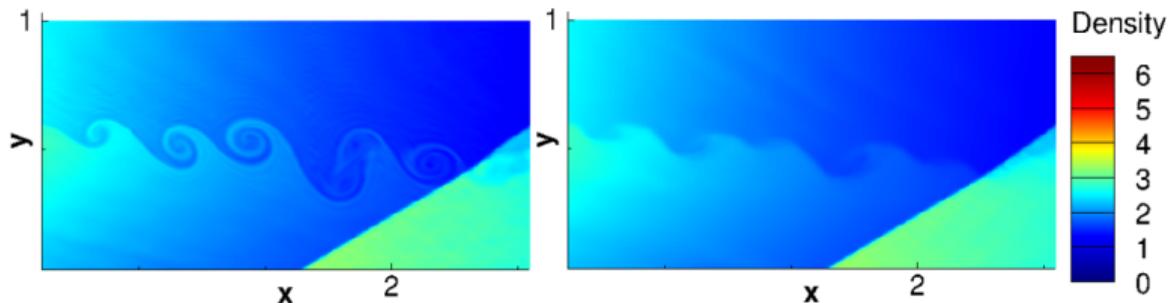
- Hybrid DG / Finite Volume operator
- Interpret solution polynomial differently
- Introduce **virtual FV grid** within each DG element
- Solve a TVD Finite volume method in troubled cells
- Keep high order accuracy wherever possible
- Switch DG2FV and vice versa  $\Rightarrow$  **Experience / Parameter tuning required**



## A DG method for shock capturing

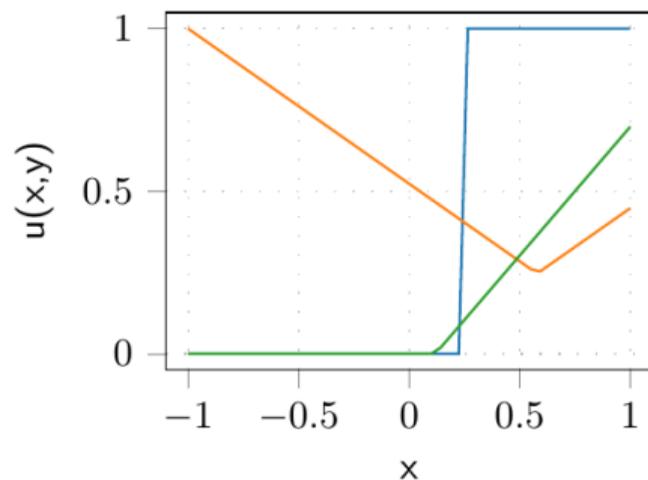
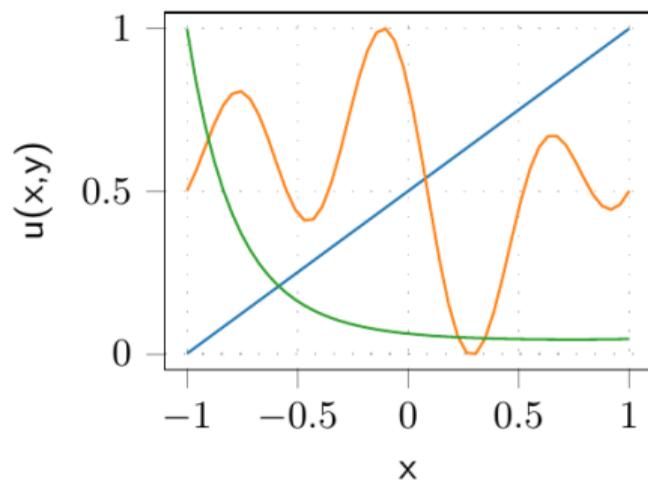


Coupled DG/FV subcell vs. pure FV:



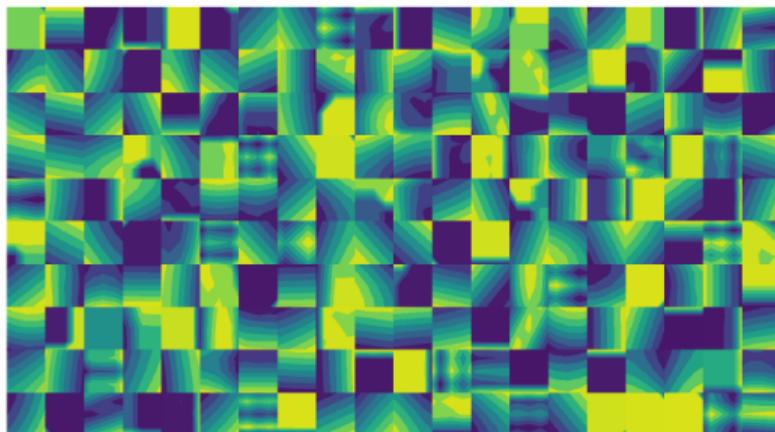
## Shock localization through ML

- General idea: Decouple the shock localization and the shock capturing to ameliorate parameter tuning
- First task: Train a **CNN-based binary classifier** on element data to detect shocks without regarding their numerical representation
- Second task: Localize the shock within an element
- Training data: Smooth and non-smooth functions

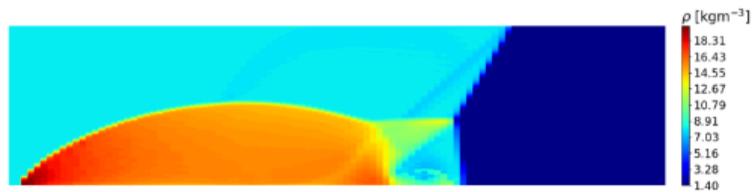


## Shock localization through ML

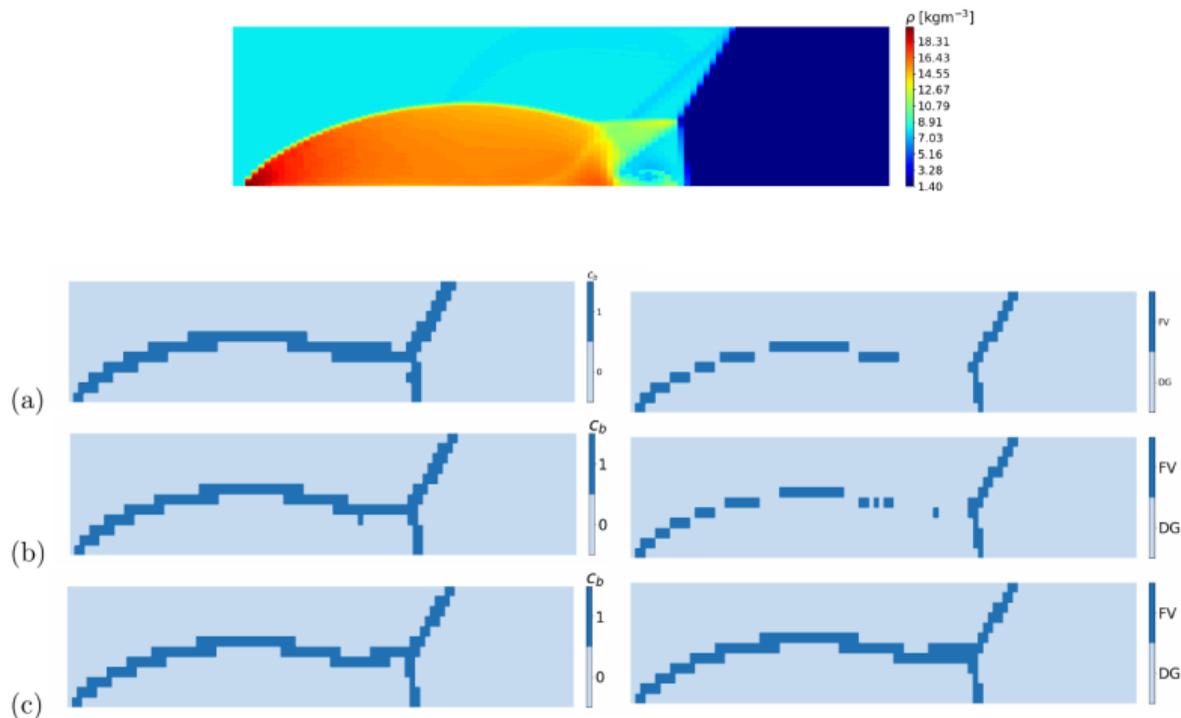
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## Shock localization through ML



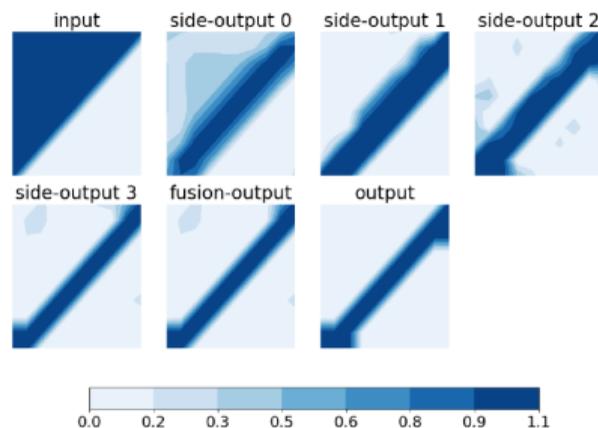
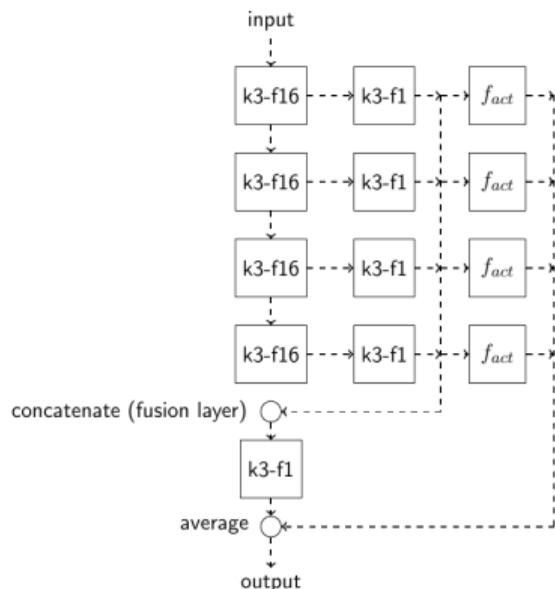
# Shock localization through ML



**Figure 4.10.:** Classification results of models  $C_{N4}$ ,  $C_{N5}$ , and  $C_{N9}$  (left) and the Jameson indicator (right) for the DMR on a mesh with 1224 elements at  $t_{end} = 0.2$ . (a)  $N = 4$ , (b)  $N = 5$ , (c)  $N = 9$ .

## Shock localization through ML

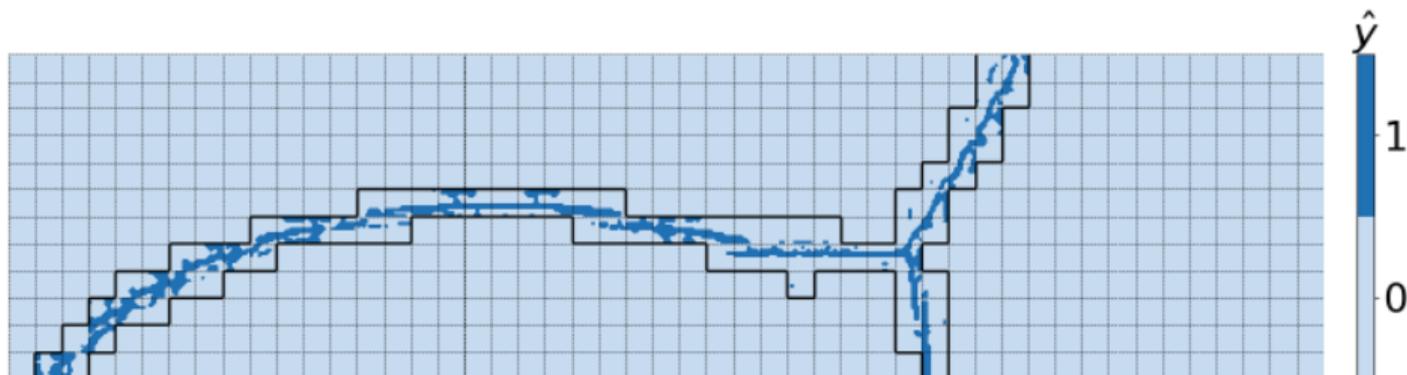
- Shocks can be safely detected by the NN indicator, without additional parameter tuning
- **Consistent detection**, not dependent on numerical scheme: not a troubled cell indicator!
- Task 2: Localize the shock within an element: **Holistic Edge Detection**



(d) Model  $C_8$  ( $N = 9$ ).

## Shock localization through ML

- Task 2: Localize the shock [within an element](#)



- Future work: [h/r adaptation](#) within the element to capture shock front sharply

# Final Thoughts

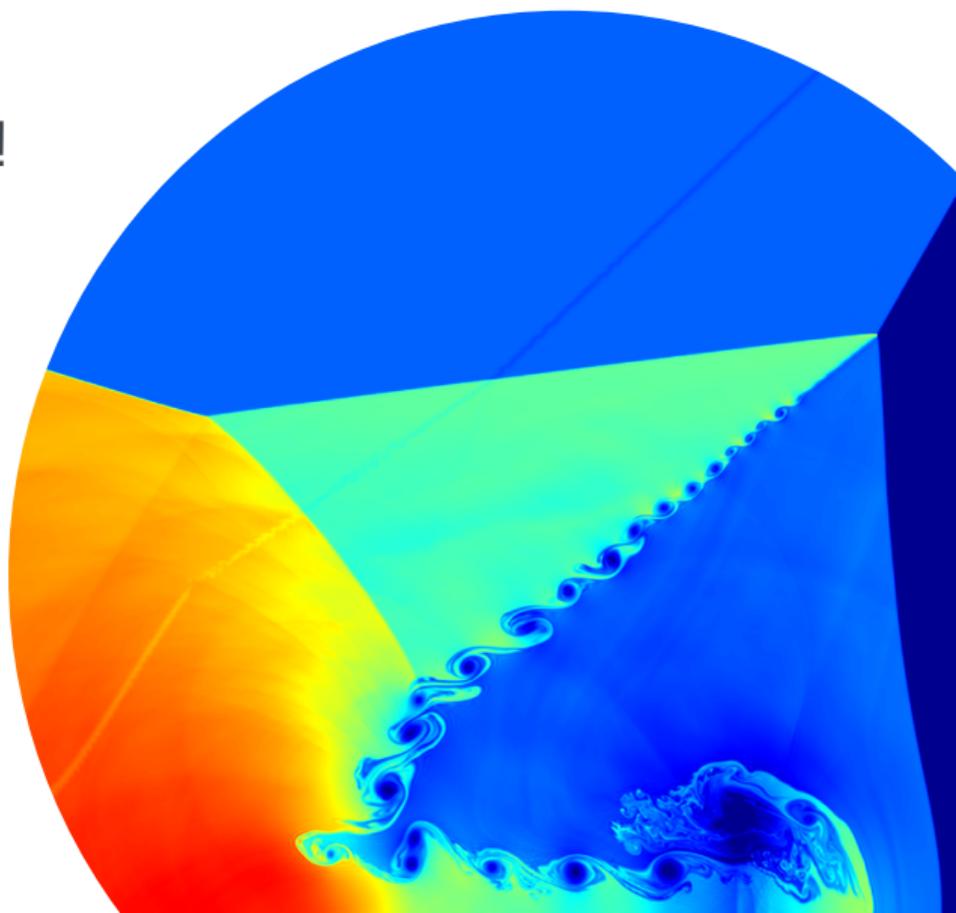
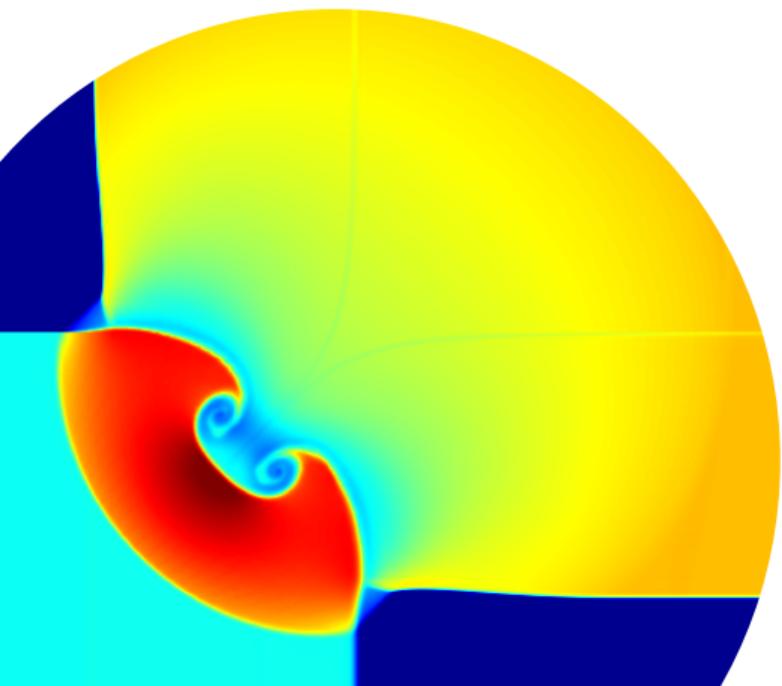
6

## Some final thoughts on data-informed models, engineering and HPC

- Machine Learning is not a **silver bullet**
- First successes: ML can help build **subscale models from data**, not just for turbulence
- A lot of representative data is needed... maybe we already have the data? Computations, experiments...
- In this work, the computational times were: DNS:  $\mathcal{O}(10^5)$  CPUh, data preparation  $\mathcal{O}(10^3)$ , Training the RNN:  $\mathcal{O}(10^1 - 10^2)$ : **Is it worth it?**
- Incorporating **physical constraints** (e.g. realizability, positivity) field of research
- Self-learning algorithms: Reinforcement learning
- "**Philosophical aspects**": Interpretability of the models and "who should learn what?"
- HPC: Training has to done on GPUs (easy for supervised learning, bit more complicated for reinforcement learning), but ...
- What about model deployment? GPU (native) or CPU (export model)?
- **Coupling** of CFD solver (Fortran) to Neural Network (python): In our case, f2py is a very cumbersome solution
- Hybrid CPU/GPU codes, or rewrite it all for the GPU?
- Data storage policy: where to compute/store the data (reproducibility)

**flexi-project.org**

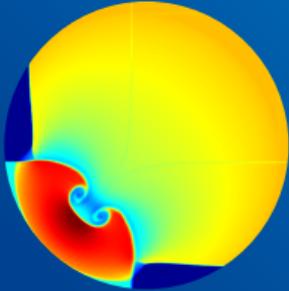
Thank you for your attention!





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